Mathematics

Question1

If $D \subseteq R$ and $f : D \rightarrow R$ defined by $f(x) = \frac{x^2+x+a}{x^2-x+a}$ is a surjection, then ' a ' lies in the interval.

Options:

A.

R

B.

$(0, \infty)$

C.

$(-\infty, 0)$

D.

$(0, 1)$

Answer: C

Solution:

$f(x) = \frac{x^2+x+a}{x^2-x+a}$

$\because f(x)$ is surjection, then range $=$ Codomain

Range $= R$

Now for range, put $f(x) = y$.

$$y = \frac{x^2+x+a}{x^2-x+a}$$

$x^2(y-1) - x(y+1) + ay - a = 0$

Now $x \in R \Rightarrow D \geq 0$

$(y+1)^2-4(y-1)(ay-a) \geq 0$
$\Rightarrow (y+1)^2-4a(y-1)^2 \geq 0, \forall y \in R$
$\Rightarrow y^2(1-4a)+y(2+8a)+1-4a \geq 0, \forall y \in R$

$\therefore 1-4a \geq 0$ and $D \leq 0$
$a \leq \frac{1}{4}$ and $(2+8a)^2-4(1-4a)^2 \leq 0$
$\Rightarrow (1+4a)^2-(1-4a)^2 \leq 0$
$\Rightarrow (1+4a+1-4a)(1+4a-1+4a) \leq 0$
$\Rightarrow 2(8a) \leq 0$
$\Rightarrow a \leq 0$
$\therefore a \in (-\infty, 0)$.

Question2

If the domain of the real valued function $f(x)=\frac{1}{\sqrt{\log_{\frac{1}{3}}\left(\frac{x-1}{2-x}\right)}}$ is (a, b), then $2b=$

Options:

A.

$a-1$

B.

a

C.

$a+1$

D.

$a+2$

Answer: D

Solution:

Given, $f(x)=\frac{1}{\sqrt{\log_{1/3}\left(\frac{x-1}{2-x}\right)}}$

$\therefore$ Domain

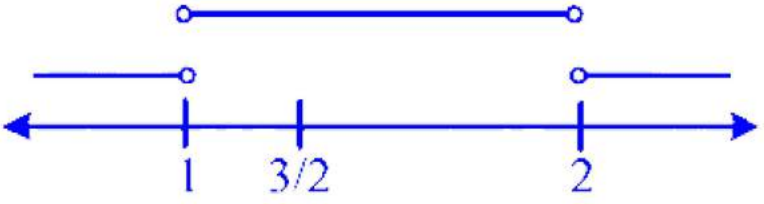
$$\log_{1/3} \frac{x-1}{2-x} > 0 \text{ and } \frac{x-1}{2-x} > 0$$

$$\Rightarrow \quad \frac{x-1}{2-x} < \left(\frac{1}{3}\right)^0 \Rightarrow \frac{x-1}{2-x} < 1$$

$$\Rightarrow \quad \frac{x-1}{2-x} - 1 < 0$$

$$\Rightarrow \quad \frac{x-1-2+x}{2-x} < 0 \Rightarrow \frac{2x-3}{2-x} < 0$$

Now,



Now, $x \in \left(1, \frac{3}{2}\right)$

$\therefore \quad a = 1, b = \frac{3}{2}$

Now, $2b = 3 = a + 2$

Question3

If $\frac{1}{2\cdot7} + \frac{1}{7\cdot12} + \frac{1}{12\cdot17} + \frac{1}{17\cdot22} + \ldots$ to 10 terms $= k$, then $k =$

Options:

A.

$\frac{2}{51}$

B.

$\frac{5}{51}$

C.

$\frac{5}{52}$

D.

$\frac{1}{26}$

Answer: C

Solution:

Given,

$$\frac{1}{2\cdot 7}+\frac{1}{7\cdot 12}+\frac{1}{12\cdot 17}+\ldots+10 \text{ terms.}$$

$$=\frac{1}{5}\left[\frac{5}{2\cdot 7}+\frac{5}{7\cdot 12}+\frac{5}{12\cdot 17}+\ldots+10 \text{ terms}\right]$$

$$=\frac{1}{5}\left[\frac{7-2}{2\cdot 7}+\frac{12-7}{7\cdot 12}+\frac{17-12}{12\cdot 17}+\ldots+10 \text{ terms}\right]$$

$$=\frac{1}{5}\left[\begin{array}{l}\frac{1}{2}-\frac{1}{7}+\frac{1}{7}-\frac{1}{12}+\left(\frac{1}{12}-\frac{1}{17}\right)+\ldots \\ +\left(\frac{1}{2+9\times 5}-\frac{1}{7+9\times 5}\right)\end{array}\right]$$

$$=\frac{1}{5}\left(\frac{1}{2}-\frac{1}{52}\right)=\frac{1}{5}\left(\frac{25}{52}\right)=\frac{5}{52}$$

Question4

If the system of simultaneous linear equations $x+\lambda y-2z=1, x-y+\lambda z=2$ and $x-2y+3z=3$ is inconsistent for $\lambda=\lambda_1$ and λ_2, then $\lambda_1+\lambda_2=$

Options:

A.

5

B.

$\sqrt{5}$

C.

1

D.

-1

Answer: C

Solution:

Given, $x+\lambda y-2z=1$

$x-y+\lambda z=2$

And $x-2y+3z=3$

$\because$ Given, system is inconsistent

$$D=0 \Rightarrow \begin{vmatrix} 1 & \lambda & -2 \\ 1 & -1 & \lambda \\ 1 & -2 & 3 \end{vmatrix}=0$$

$\Rightarrow 1(-3+2\lambda)-\lambda(3-\lambda)-2(-2+1)=0$

$\Rightarrow 2\lambda-3-3\lambda+\lambda^2+2=0$

$$\lambda^2-\lambda-1=0 \begin{cases} \lambda_1 \\ \lambda_2 \end{cases}$$

$\therefore \lambda_1+\lambda_2=1$

Question5

The system of linear equation $(\sin\theta)x+y-2z=0$, $2x-y+(\cos\theta)z=0$ and $-3x+(\sec\theta)y+3z=0$, where $\theta \neq (2n+1)\frac{\pi}{2}$, has non-trivial solution for

Options:

A.

no value of θ

B.

$\theta=n\pi+\frac{\pi}{4}, n \in Z$

C.

$\theta=\tan^{-1}\left(\frac{3}{4}\right)$

D.

$\theta=\tan^{-1}\left(\frac{4}{3}\right)$

Answer: A

Solution:

Given, $(\sin\theta)x+y-2z=0$

$2x-y+(\cos\theta)z=0$

And $-3x+(\sec\theta)y+3z=0$

∵ system has non-trivial solution.

$$\Delta = 0$$
$$\begin{vmatrix} \sin\theta & 1 & -2 \\ 2 & -1 & \cos\theta \\ -3 & \sec\theta & 3 \end{vmatrix} = 0$$
$$\sin\theta(-3-1) - 1(6+3\cos\theta) - 2(2\sec\theta - 3) = 0$$
$$\Rightarrow -4\sin\theta - 6 - 3\cos\theta - 4\sec\theta + 6 = 0$$
$$\Rightarrow 4\sec\theta + 3\cos\theta + 4\sin\theta = 0$$
$$\Rightarrow 4 + 3\cos^2\theta + 4\sin\theta\cos\theta = 0$$
$$\Rightarrow 4 + 3\left(\frac{1+\cos 2\theta}{2}\right) + 2\sin 2\theta = 0$$
$$\Rightarrow 8 + 3 + 3\cos 2\theta + 4\sin 2\theta = 0$$
$$\Rightarrow 11 + 3\cos 2\theta + 4\sin 2\theta = 0$$

This equation has no solution because the maximum value of $4\sin 2\theta + 3\cos 2\theta$ is 5 . Since $5 + 11 \neq 0$, there is no value of θ that satisfies the equation.

Question6

If $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$**, then** $\operatorname{adj}(\operatorname{adj}(\operatorname{adj} A))$

Options:

A.

A

B.

A^{-1}

C.

$|A| A^{-1}$

D.

$\frac{A^{-1}}{|A|}$

Answer: C

Solution:

$$A=\begin{bmatrix}1 & 2\\ 3 & 4\end{bmatrix}$$
$\because \operatorname{adj}(\operatorname{adj} A)=|A|^{n-2}A$
$\operatorname{adj}(\operatorname{adj}(\operatorname{adj} A))=|\operatorname{adj} A|^{2-2}\cdot \operatorname{adj} A$
$=\operatorname{adj} A=|A|A^{-1}\left(\because A^{-1}=\frac{1}{|A|}\operatorname{adj} A\right)$

Question7

The sum of all the roots of the equation

$$\begin{vmatrix} x & -3 & 2\\ -1 & -2 & (x-1)\\ 1 & (x-2) & 3\end{vmatrix}=0 \text{ is}$$

Options:

A.

13

B.

3

C.

2

D.

7

Answer: B

Solution:

Given,

$$\begin{vmatrix} x & -3 & 2\\ -1 & -2 & x-1\\ 1 & x-2 & 3\end{vmatrix}=0$$

$\Rightarrow x(-6-(x-1)(x-2))+3(-3-(x-1))+2(-(x-2)+2)=0$

$\Rightarrow x\left(-6-x^2+3x-2\right)+3(-3-x+1)+2(-x+2+2)=0$
$\Rightarrow x\left(-x^2+3x-8\right)+3(-2-x)+2(4-x)=0$
$\Rightarrow -x^3+3x^2-8x-6-3x+8-2x=0$
$\therefore$ Sum of roots $=\frac{-3}{-1}=3.$

Question8

One of the values of $\sqrt{24-70i}+\sqrt{-24+70i}$ is

Options:

A.

$2+12i$

B.

$12-2i$

C.

$-12+2i$

D.

$-12-2i$

Answer: D

Solution:

We have,

$$\begin{aligned}
z &= \sqrt{24-70i}+\sqrt{-24+70i} \\
z^2 &= 24-70i+(-24+70i) \\
&\quad +2\sqrt{24-70i}\sqrt{-24+70i} \\
&= 2i(24-70i)=2(24i+70) \\
&= 4(12i+35) \\
z &= \sqrt{4}\sqrt{35+12i} \\
&= 2\left(\pm\left(\frac{\operatorname{Re} z+|z|}{2}\right)^{1/2}\right. \\
&\quad \left.+i\left(\frac{-\operatorname{Re} z+|z|}{2}\right)^{1/2}\right) \\
&= \pm 2\left(\left(\frac{35+37}{2}\right)^{1/2}+i\left(\frac{-35+37}{2}\right)^{1/2}\right) \\
&= \pm 2(6+i)=\pm(12\pm 2i)
\end{aligned}$$

Question9

The set of all values of θ such that $\frac{1-i\cos\theta}{1+2i\sin\theta}$ is purely imaginary is

Options:

A.

$\{n\pi+(-1)^n\frac{\pi}{4}, n\in z\}$

B.

$\{\frac{n\pi}{2}+(-1)^n\frac{\pi}{4}, n\in z\}$

C.

$\{n\pi+(-1)^n\frac{\pi}{2}, n\in z\}$

D.

$\{2n\pi\pm\frac{\pi}{4}, n\in z\}$

Answer: B

Solution:

$z=\frac{1-i\cos\theta}{1+2i\sin\theta}$ is purely imaginary

$$\therefore z+\bar{z}=0$$
$$\Rightarrow \frac{1-i\cos\theta}{1+2i\sin\theta}+\frac{1+i\cos\theta}{1-2i\sin\theta}=0$$
$$\frac{\begin{array}{l}1-2i\sin\theta-i\cos\theta-2\sin\theta\cos\theta+1\\+i\cos\theta+2i\sin\theta-2\sin\theta\cos\theta\end{array}}{(1+2i\sin\theta)(1-2i\sin\theta)}=0$$
$$\Rightarrow (2-4\sin\theta\cos\theta)=0$$
$$\Rightarrow 2\sin 2\theta=2\Rightarrow \sin 2\theta=1$$
$$\Rightarrow \sin 2\theta=\sin\frac{\pi}{2}$$
$$\therefore \quad 2\theta=n\pi+(-1)^n\frac{\pi}{2}, n\in z$$
$$\therefore \quad \theta=\frac{n\pi}{2}+(-1)^n\frac{\pi}{4}, n\in z.$$

Question10

If $\cos\alpha+\cos\beta+\cos\gamma=0=\sin\alpha+\sin\beta+\sin\gamma$, then $\sin 2\alpha+\sin 2\beta+\sin 2\gamma=$

Options:

A.

$\cos(\alpha+\beta)+\cos(\beta+\gamma)+\cos(\gamma+\alpha)$

B.

$\cos^2\alpha+\cos^2\beta+\cos^2\gamma$

C.

$\sin^2\alpha+\sin^2\beta+\sin^2\gamma$

D.

$\cos(2\alpha-\beta-\gamma)+\cos(2\beta-\gamma-\alpha)+\cos(2\gamma-\alpha-\beta)$

Answer: A

Solution:

We have

$$\cos\alpha+\cos\beta+\cos\gamma=0$$
$$=\sin\alpha+\sin\beta+\sin\gamma$$

Let $a=\cos\alpha+i\sin\alpha$

$$b=\cos\beta+i\sin\beta$$
$$c=\cos\gamma+i\sin\gamma$$
$$\therefore a+b+c=(\cos\alpha+\cos\beta+\cos\gamma)$$
$$+i(\sin\alpha+\sin\beta+\sin\gamma)$$

$a+b+c=0$

and $a^{-1}+b^{-1}+c^{-1}=0$

$$\therefore ab+bc+ca=0$$
$$\Rightarrow \cos(\alpha+\beta)+\cos(\beta+\gamma)$$
$$+\cos(\gamma+\alpha)=0$$

Now $a^2+b^2+c^2=(a+b+c)^2$

$-2(ab+bc+ca)$

$\therefore \sin 2\alpha+\sin 2\beta+\sin 2\gamma=0$

Question11

If α is a root of the equation $x^2 - x + 1 = 0$, then

$\left(\alpha + \frac{1}{\alpha}\right)^3 + \left(\alpha^2 + \frac{1}{\alpha^2}\right)^3 + \left(\alpha^3 + \frac{1}{\alpha^3}\right)^3 + \left(\alpha^4 + \frac{1}{\alpha^4}\right)^3 + \ldots$ to 12 terms $=$

Options:

A.

-32

B.

32

C.

0

D.

16

Answer: C

Solution:

$x^2 - x + 1 = 0 \rightarrow \alpha$
$\therefore \alpha = -\omega \Rightarrow \alpha^3 = -\omega^3 = -1$

Now, $\left(\alpha + \frac{1}{\alpha}\right)^3 + \left(\alpha^2 + \frac{1}{\alpha^2}\right)^3 + \ldots$ to 12 terms

$= \left(\frac{\alpha^2+1}{\alpha}\right)^3 + \left(\frac{\alpha^4+1}{\alpha^2}\right)^3 + (-1-1)^3 + \ldots + 12$ terms
$= \left[(1)^3 + (1)^3 + (-2)^3 + (-1)^3 + (-1)^3 + (2)^3\right]2 = 0$

Question12

If the equations $x^2 + px + 2 = 0$ and $x^2 + x + 2p = 0$ have a common root, then the sum of the roots of the equation $x^2 + 2px + 8 = 0$ is

Options:

A.

-3

B.

3

C.

6

D.

-6

Answer: C

Solution:

$$x^2 + px + 2 = 0$$
$$x^2 + x + 2p = 0$$

Let ' α ' be the common root

$$a^2 + pa + 2 = 0$$
$$a^2 + a + 2p = 0$$
$$- \quad - \quad - \quad \quad -$$
$$\overline{a(p-1) + 2 - 2p = 0}$$

$$\therefore \quad \alpha = 2$$
$$4 + 2p + 2 = 0$$
$$2p = -6$$
$$p = -3$$

$\therefore$ Sum of roots of $x^2 + 2px + 8 = 0$ (i.e., $x^2 - 6x + 8 = 0$) is 6 .

Question13

If both roots of the equation $x^2 - 5ax + 6a = 0$ exceed 1 , then the range of ' a ' is

Options:

A.

$[-1, 0) \cup \left[\frac{24}{25}, \infty\right)$

B.

$\left[\frac{24}{25}, \infty\right)$

C.

$[-1, 0)$

D.

R

Answer: B

Solution:

$x^2 - 5ax + 6a = 0$

$\because$ Both roots are greater than 2 .

$$D \geq 0, \frac{-b}{2a} > 1, f(0) > 0$$
$$(5a)^2 - 4(1)(6a) \geq 0, \frac{5a}{2} > 1, 6a > 0$$
$$\Rightarrow a(25a - 24) \geq 0, a > \frac{2}{5}, a > 0$$
$$\therefore a \in \left[\frac{24}{25}, \infty\right)$$

Question14

If α, β, γ and δ are the roots of the equation $x^4 - 4x^3 + 3x^2 + 2x - 2 = 0$ such that α and β are integers and γ, δ are irrational numbers, then $\alpha + 2\beta + \gamma^2 + \delta^2 =$

Options:

A.

5

B.

7

C.

11

D.

13

Answer: C

Solution:

$$x^4 - 4x^3 + 3x^2 + 2x - 2 = 0 \begin{cases} \alpha \\ \beta \\ \gamma \\ \delta \end{cases}$$

$$\Rightarrow x^3(x-1) - 3x^2(x-1) + 2(x-1) = 0$$
$$\Rightarrow (x-1)\left(x^3 - 3x^2 + 2\right) = 0$$

$$\Rightarrow (x-1)\left[x^2(x-1) - 2x(x-1) - 2(x-1)\right] = 0$$
$$\Rightarrow (x-1)(x-1)\left(x^2 - 2x - 2\right) = 0$$
$$\therefore \quad \alpha = 1, \gamma + \delta = 2$$
$$\beta = 1, \gamma\delta = -2$$
$$\therefore \alpha + 2\beta + \gamma^2 + \delta^2$$

$$= 1 + 2 + (\gamma + \delta)^2 - 2\gamma\delta$$
$$= 3 + 4 + 4 = 11$$

Question15

The equation having the multiple root of the equation $x^4 + 4x^3 - 16x - 16 = 0$ as its roots is

Options:

A.

$x^2 + 2x - 3 = 0$

B.

$x^2 - 3x + 2 = 0$

C.

$x^2 + x - 2 = 0$

D.

$x^2-4x+3=0$

Answer: C

Solution:

We have, $x^4+4x^3-16x-16=0$

$$\begin{aligned}
&\Rightarrow x^3(x-2)+6x^2(x-2)+12x(x-2)+8(x-2)=0\\
&\Rightarrow (x-2)\left(x^3+6x^2+12x+8\right)=0\\
&\Rightarrow (x-2)\left[x^2(x+2)+4x(x+2)+4(x+2)\right]=0\\
&\Rightarrow (x-2)(x+2)\left(x^2+4x+4\right)=0\\
&\Rightarrow (x-2)(x+2)^3=0
\end{aligned}$$

∴ Multiply roots is -2 .

(a) $x^2+2x-3=0 \Rightarrow (x+3)(x-1)=0$

(b) $x^2-3x+2=0 \Rightarrow (x-2)(x-1)=0$

(c) $x^2+x-2=0 \Rightarrow (x+2)(x-1)=0$

(d) $x^2-4x+3=0 \Rightarrow (x-3)(x-1)=0$

Question16

There are 15 stations on a train route and the train has to be stopped at exactly 5 stations among these 15 stations. If it stops at atleast two consecutive stations, then the number of ways in which the train can be stopped is

Options:

A.

${}^{11}C_5$

B.

${}^{15}C_5$

C.

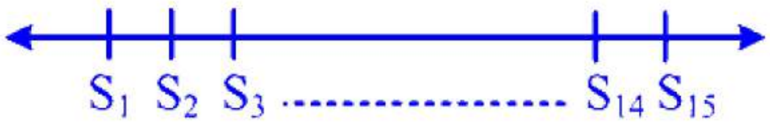
${}^{15}C_5-{}^{11}C_5$

D.

${}^{15}C_{10}-{}^{9}C_5$

Answer: C

Solution:



$\because$ Total number of ways in which train has stopped at exactly 5 stations $= {}^{15}C_5$

And total number of ways in which train has not stopped in consecutive stations

$x_1 + x_2 + x_3 + x_4 + x_5 + L = 15 - 4 = 11$

$x_i =$ number of station left before i^{th} station

$x_1 \geq 0 x_2, x_3, x_4, x_5 \geq 1$, and $L =$ station

left after 4 stations

$$x_1 + (x_2 - 1) + (x_3 - 1) + (x_4 - 1) + (x_5 - 1) + L \\ = 11 - 4 = 7$$

$\therefore$ Total number of solution $= {}^{7+5-1}C_{6-1}$
$= {}^{11}C_5$

$\therefore$ Required ways $= {}^{15}C_5 - {}^{11}C_5$

Question17

Number of all possible ways of distributing eight identical apples among three persons is

Options:

A.

45

B.

42

C.

39

D.

36

Answer: A

Solution:

Number of all possible ways of distributing eight identical apples among 3 persons is number of non-negative integral solution of equation is

$$x + y + z = 8$$
$$\therefore {}^{8+3-1}C_{3-1}$$
$$= {}^{10}C_2$$
$$= \frac{10 \times 9}{2 \times 1} = 45$$

Question18

Number of all possible words (with or without meaning) that can be formed using all the letters of the word CABINET in which neither the word CAB nor the word NET appear is

Options:

A.

5040

B.

4806

C.

4800

D.

5034

Answer: B

Solution:

The number of word in which
"CAB" appear is CAB <u>I</u> <u>N</u> <u>E</u> <u>T</u> = 5!
The number of word in which
"NET" appear is NET <u>C</u> <u>A</u> <u>B</u> <u>T</u> = 5!

The number of words in CAB and NET both appears = CAB NET $I = 3$!

$\therefore$ Required number of ways

$$\begin{aligned} &= 7! - (5! + 5! - 3!) \\ &= 5040 - (120 + 120 - 6) \\ &= 5040 - 234 \\ &= 4806 \end{aligned}$$

Question19

Numerically greatest term in the expansion of $(2x - 3y)^n$ when $x = \frac{7}{2}, y = \frac{3}{7}$ and $n = 13$ is

Options:

A.

$13 \cdot 3^5 \cdot 7^9$

B.

$13 \cdot 3^4 \cdot 7^9$

C.

$26 \cdot 3^5 \cdot 7^9$

D.

$26 \cdot 3^4 \cdot 7^9$

Answer: C

Solution:

For numerically greatest term in the expansion of $(2x - 3y)^n$

$$\begin{aligned} r &= \frac{n+1}{1 + \left|\frac{2x}{3y}\right|} = \frac{13+1}{1 + \left|\frac{2 \times \frac{7}{2}}{3 \times \frac{3}{7}}\right|} = \frac{14}{1 + \frac{49}{9}} \\ &= \frac{14 \times 9}{9 + 49} = \frac{126}{58} \end{aligned}$$

$\therefore \quad [r] = 2$

$\therefore T_3$ will be numerically greatest term

$$\therefore \quad T_3 = {}^{13}C_2(2x)^{11}(-3y)^2$$
$$= \frac{13 \times 12}{2} \times \left(2 \times \frac{7}{2}\right)^{11}\left(-3 \times \frac{3}{7}\right)^2$$
$$= 13 \times 6 \times 7^{11} \times \frac{3^4}{7^2}$$
$$= 263^5 7^9$$

Question20

If $C_0, C_1, C_2, \ldots, C_8$ are the binomial coefficients in the expansion of $(1+x)^8$, then $\sum_{r=1}^{8} r^3 \frac{C_r}{C_{r-1}} =$

Options:

A.

540

B.

336

C.

105

D.

270

Answer: A

Solution:

$$\sum_{r=1}^{8} r^3 \cdot \frac{C_r}{C_{r-1}} = \sum_{r=1}^{8} r^3 \cdot \frac{8-r+1}{r}$$
$$= \sum_{r=1}^{8} (9r^2 - r^3)\left[\because \frac{{}^nC_r}{{}^nC_{r-1}} = \frac{n-r+1}{r}\right]$$
$$= 9\left(\frac{8}{6}(8+1)(17)\right) - \left(\frac{8}{2} \times 9\right)^2$$
$$= 3 \times 4 \times 9 \times 17 - 4 \times 4 \times 9 \times 9$$
$$= 4 \times 9(51-36) = 36 \times 15 = 540$$

Question21

If $\frac{x+3}{(x+1)(x^2+2)} = \frac{a}{x+1} + \frac{bx+c}{x^2+2}$**, then** $a - b + c =$

Options:

A.

0

B.

1

C.

3

D.

2

Answer: C

Solution:

$$\frac{x+3}{(x+1)(x^2+2)} = \frac{a}{x+1} + \frac{bx+c}{x^2+2}$$
$$\Rightarrow x+3 = a\left(x^2+2\right) + (bx+c)(x+1)$$
put $x = -1$
$$2 = a(3) \Rightarrow a = \frac{2}{3}$$

put $x = 0$

$3 = 2a + c$
$c = 3 - \frac{4}{3} = \frac{5}{3}$

put $x = 1$

$$4 = 3a + (b+c)2$$
$$4 = 2 + \left(b + \frac{5}{3}\right)2$$
$$1 = b + \frac{5}{3} \Rightarrow b = \frac{-2}{3}$$
$$\therefore a - b + c = \frac{2}{3} + \frac{2}{3} + \frac{5}{3} = \frac{9}{3} = 3$$

Question22

If $3\sin\theta + 4\cos\theta = 3$ and $\theta \neq (2n+1)\frac{\pi}{2}$, then $\sin 2\theta =$

Options:

A.

$\frac{336}{625}$

B.

$-\frac{7}{25}$

C.

$\frac{24}{25}$

D.

$-\frac{336}{625}$

Answer: D

Solution:

$3\sin\theta + 4\cos\theta = 3$

On squaring both sides, we get $79\sin^2\theta + 16\cos^2\theta + 24\sin\theta\cos\theta = 9$

$$\begin{aligned}
&\Rightarrow 9 + 7\cos^2\theta + 24\sin\theta\cos\theta = 9 \\
&\Rightarrow \cos\theta(7\cos\theta + 24\sin\theta) = 0 \\
&\Rightarrow \cos\theta \neq 0 \\
&\quad \theta \neq (2n+1)\frac{\pi}{2} \\
&\text{and } 7\cos\theta + 24\sin\theta = 0
\end{aligned}$$

$$\tan\theta = \frac{-7}{24}$$

$$\begin{aligned}
\therefore \sin 2\theta &= \frac{2\tan\theta}{1+\tan^2\theta} \\
&= \frac{2\times\left(\frac{-7}{24}\right)}{1+\frac{49}{576}} = \frac{-14\times 24}{625} \\
&= \frac{-336}{625}
\end{aligned}$$

Question23

$$\frac{\cos 15^\circ \cos^2 22\frac{1}{2}^\circ - \sin 75^\circ \sin^2 \cdot 52\frac{1}{2}^\circ}{\cos^2 15^\circ - \cos^2 75^\circ}$$

Options:

A.

1

B.

$\frac{1}{2}$

C.

$\frac{1}{4}$

D.

$\frac{1}{8}$

Answer: C

Solution:

$$\frac{\cos 15^\circ \cos^2 22\frac{1}{2}^\circ - \sin 75^\circ \sin^2 52\frac{1}{2}^\circ}{\cos^2 15^\circ - \cos^2 75^\circ}$$

$$= \frac{\cos 15^\circ \left(\cos^2 22\frac{1}{2}^\circ - \sin^2 52\frac{1}{2}^\circ\right)}{\cos^2 15^\circ - \sin^2 15^\circ} \left(\begin{matrix} \because \sin 75^\circ = \sin(90^\circ - 15^\circ) \\ = \cos 15^\circ \end{matrix}\right)$$

$$= \frac{\cos 15^\circ \left(\frac{1+\cos 45^\circ}{2} - \frac{1-\cos 105^\circ}{2}\right)}{\cos 30^\circ}$$

$$= \frac{\cos 15^\circ}{2} \frac{[\cos 45^\circ + \cos(90 + 15^\circ)]}{\frac{\sqrt{3}}{2}}$$

$$= \frac{1}{\sqrt{3}}(\cos 15^\circ \cos 45^\circ - \sin 15^\circ \cos 15^\circ)$$

$$= \frac{1}{2\sqrt{3}}(2\cos 15^\circ \cos 45^\circ - 2\cos 15^\circ \sin 15^\circ)$$

$$= \frac{1}{2\sqrt{3}}[\cos(60^\circ) + \cos 30^\circ - \sin 30^\circ]$$

$$= \frac{1}{2\sqrt{3}}\left(\frac{\sqrt{3}}{2}\right) = \frac{1}{4}$$

Question24

$16 \sin 12^{\circ} \cos 18^{\circ} \sin 48^{\circ} =$

Options:

A.

$\sqrt{10-2\sqrt{5}}$

B.

$\sqrt{10+2\sqrt{5}}$

C.

$\sqrt{5}-1$

D.

$\sqrt{5}+1$

Answer: A

Solution:

$16 \sin 12^{\circ} \cos 18^{\circ} \sin 48^{\circ}$

$$\begin{aligned}
&= 16 \sin 12^{\circ} \sin 72^{\circ} \sin 48^{\circ} \\
&= 16 \sin 12^{\circ} \sin (60+12^{\circ}) \sin (60^{\circ}-12^{\circ}) \\
&= \frac{16}{4} \sin 3(12^{\circ}) = 4 \sin 36^{\circ} \\
&= \frac{4\sqrt{10-2\sqrt{5}}}{4} = \sqrt{10-2\sqrt{5}}
\end{aligned}$$

Question25

Number of solutions of the equation $\sin^2\theta + 2\cos^2\theta - \sqrt{3}\sin\theta\cos\theta = 2$ lying in the interval $(-\pi, \pi)$ is

Options:

A.

2

B.

3

C.

4

D.

5

Answer: B

Solution:

$\sin^2\theta + 2\cos^2\theta - \sqrt{3}\sin\theta\cos\theta = 2$

$\Rightarrow 1 + \cos^2\theta - \sqrt{3}\sin\theta\cos\theta = 2$
$\Rightarrow 1 + \cos 2\theta - \sqrt{3}\sin 2\theta - 2 = 0$
$\Rightarrow \cos 2\theta - \sqrt{3}\sin 2\theta = 1$

Divide by $\sqrt{1^2 + (-\sqrt{3})^2} = \sqrt{4} = 2$ on both sides

$\Rightarrow \frac{1}{2}\cos 2\theta - \frac{\sqrt{3}}{2}\sin 2\theta = \frac{1}{2}$

$$\Rightarrow \quad \cos\left(2\theta + \frac{\pi}{3}\right) = \frac{1}{2}$$
$$\Rightarrow \quad 2\theta + \frac{\pi}{3} = 2n\pi \pm \frac{\pi}{3}, n \in Z$$

If $2\theta + \frac{\pi}{3} = 2n\pi + \frac{\pi}{3} \Rightarrow \theta = n\pi, n \in Z$

If $2\theta + \frac{\pi}{3} = 2n\pi - \frac{\pi}{3} \Rightarrow \theta = n\pi - \frac{\pi}{3}, n \in Z$

For $\theta = n\pi$, if $n = 0, \theta = 0 \in (-\pi, \pi)$

For $\theta = n\pi - \frac{\pi}{3}$, if $n = 0, \theta = \frac{-\pi}{3} \in (-\pi, \pi)$

if $n = 1, \theta = \pi - \frac{\pi}{3} = \frac{2\pi}{3} \in (-\pi, \pi)$

$\therefore$ Solutions are $0, \frac{-\pi}{3}, \frac{2\pi}{3}$

$\Rightarrow$ Number of solutions $= 3$

Question26

If $0 \le x < \frac{3}{4}$, then the number of values of x satisfying the equation $\tan^{-1}(2x-1) + \tan^{-1} 2x = \tan^{-1} 4x - \tan^{-1}(2x+1)$ is

Options:

A.

0

B.

1

C.

2

D.

3

Answer: B

Solution:

$$\tan^{-1}(2x-1)+\tan^{-1}2x$$
$$=\tan^{-1}4x-\tan^{-1}(2x+1)$$
$$\Rightarrow \tan^{-1}(2x-1)+\tan^{-1}(2x+1)$$
$$=\tan^{-1}4x-\tan^{-1}2x$$

$$\Rightarrow \tan^{-1}\left(\frac{2x-1+2x+1}{1-(2x-1)(2x+1)}\right)$$
$$=\tan^{-1}\left(\frac{4x-2x}{1+(4x)(2x)}\right)$$
$$\Rightarrow \tan^{-1}\left(\frac{4x}{1-4x^2+1}\right)=\tan^{-1}\left(\frac{2x}{1+8x^2}\right)$$
$$\therefore \frac{4x}{2-4x^2}=\frac{2x}{1+8x^2}$$
$$x=0$$
$$1-x^2=1+8x^2$$
$$x^2=0\Rightarrow x=0$$

$\therefore$ Only one solution.

Question27

If $\sinh^{-1}x=\cosh^{-1}y=\log(1+\sqrt{2})$**, then** $\tan^{-1}(x+y)$

Options:

A.

$67\frac{1}{2}^{\circ}$

B.

75°

C.

$22\frac{1}{2}^{\circ}$

D.

15°

Answer: A

Solution:

$$\sinh^{-1} x = \cosh^{-1} y = \log(1+\sqrt{2})$$
$$\sinh^{-1} x = \log(1+\sqrt{2})$$
$$\log\left(x+\sqrt{x^2+1}\right) = \log(1+\sqrt{2})$$
$$x = 1$$
$$\text{And } \cosh^{-1} y = \log(1+\sqrt{2})$$
$$\Rightarrow \log\left(y+\sqrt{y^2-1}\right) = \log(1+\sqrt{2})$$
$$\therefore \quad y = \sqrt{2}$$

Now, $\tan^{-1}(x+y)$

$$= \tan^{-1}(1+\sqrt{2})$$
$$= 67.5^{\circ}$$

Question28

In a $\triangle ABC$, if $c^2-a^2=b(\sqrt{3}c-b)$ and $b^2-a^2=c(c-a)$ then, $\angle ABC$

Options:

A.

30°

B.

60°

C.

45°

D.

90°

Answer: D

Solution:

$$c^2 - a^2 = \sqrt{3}bc - b^2$$
$$\frac{b^2 + c^2 - a^2}{2bc} = \frac{\sqrt{3}bc}{2bc}$$
$$\cos A = \frac{\sqrt{3}}{2}$$
$$\Rightarrow \quad A = 30^\circ$$
$$b^2 - a^2 = c^2 - ac$$
$$c^2 + a^2 - b^2 = ac$$
$$\frac{c^2+a^2-b^2}{2ac} = \frac{1}{2}$$
$$\cos B = \frac{1}{2}$$
$$\Rightarrow \quad B = 60^\circ$$
$$\because \quad \angle A + \angle B + \angle C = 180^\circ$$
$$\angle C = 180^\circ - 30^\circ - 60^\circ = 90^\circ$$

Question29

Let ABC be a triangle right angled at B. If $a = 13$ and $c = 84$, then $r + R =$

Options:

A.

42.5

B.

169

C.

98

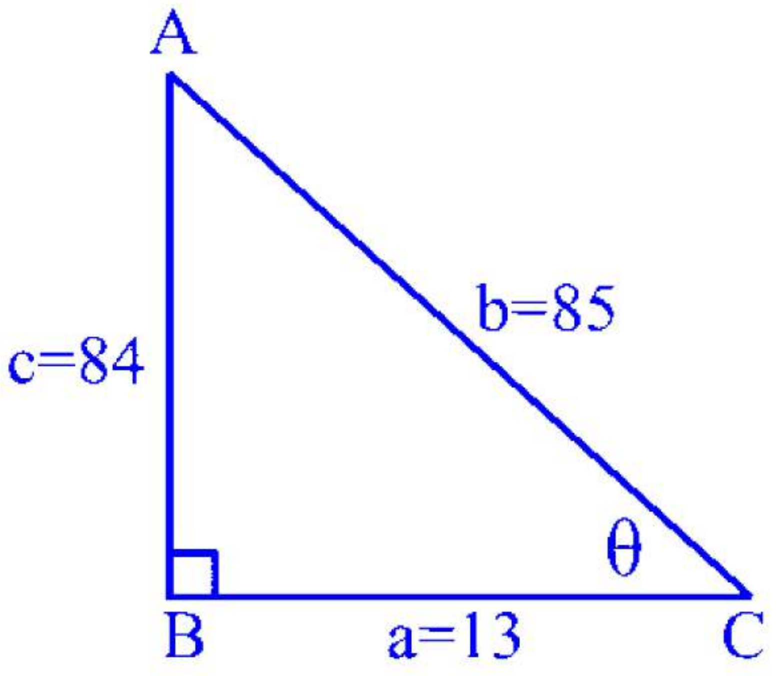
D.

48.5

Answer: D

Solution:

Given, $\angle B = 90°, a = 13, c = 84$



$$\begin{aligned} b^2 &= a^2 + c^2 \\ b^2 &= 7225 \\ b &= 85 \\ \therefore \quad r &= \frac{\Delta}{s} \\ &= \frac{\frac{1}{2} \times 13 \times 84}{\frac{13+84+85}{2}} \\ &= \frac{13 \times 84}{182} \\ &= \frac{13 \times 6}{13} = 6 \\ R &= \frac{b}{2} = \frac{85}{2} = 42.5 \end{aligned}$$

$\therefore r + R = 6 + 425 = 48.5$

Question30

If a $= (x + 2y - 3)\hat{\mathbf{i}} + (2x - y + 3)\hat{\mathbf{j}}$ and b $= (3x - 2y)\hat{\mathbf{i}} + (x - y + 1)\hat{\mathbf{j}}$ are two vectors such that a $=$ 2b, then $y - 5x =$

Options:

A.

10

B.

-10

C.

8

D.

-8

Answer: C

Solution:

Given,

$$\mathbf{a} = (x + 2y - 3)\hat{\mathbf{i}} + (2x - y + 3)\hat{\mathbf{j}}$$
$$\mathbf{b} = (3x - 2y)\hat{\mathbf{i}} + (x - y + 1)\hat{\mathbf{j}}$$
$$\because \mathbf{a} = 2\mathbf{b}$$
$$\therefore x + 2y - 3 = 2(3x - 2y)$$
$$x + 2y - 3 = 6x - 4y$$
$$5x - 6y + 3 = 0 \qquad \ldots (i)$$

$$\text{And} \quad 2x - y + 3 = 2(x - y + 1)$$
$$2x - y + 3 = 2x - 2y + 2$$
$$y = -1 \quad (Using \ \ Eq. (i))$$
$$\therefore \quad x = \frac{-9}{5}$$
$$\therefore \quad y - 5x$$
$$= -1 + 9 = 8$$

Question31

$7\hat{\mathbf{i}} - 4\hat{\mathbf{j}} + 7\hat{\mathbf{k}}, \hat{\mathbf{i}} - 6\hat{\mathbf{j}} + 10\hat{\mathbf{k}}, -\hat{\mathbf{i}} - 3\hat{\mathbf{j}} + 4\hat{\mathbf{k}}, 5\hat{\mathbf{i}} - \hat{\mathbf{j}} + \hat{\mathbf{k}}$ are the position vectors of the points A, B, C and D respectively. If $p\hat{\mathbf{i}} + q\hat{\mathbf{j}} + r\hat{\mathbf{k}}$ is the position vector of the point of intersection of the diagonals of the quadrilateral $ABCD$, then $p + q + r =$

Options:

A.

4

B.

5

C.

0

D.

1

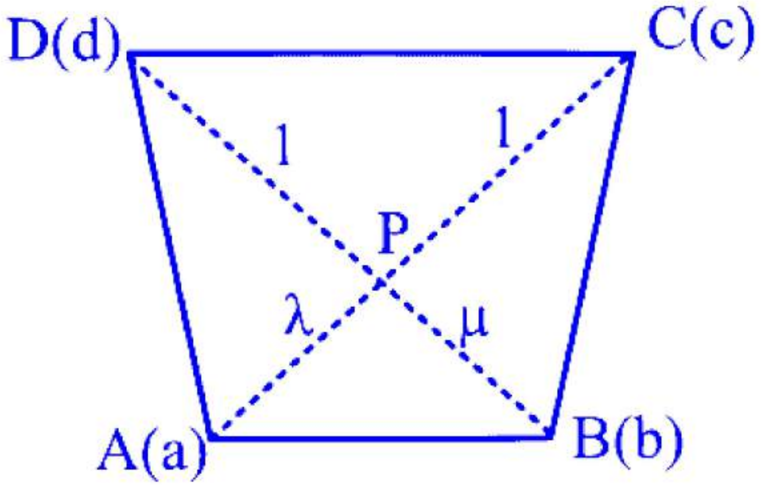
Answer: B

Solution:

Position vectors of A, B, C and D are

$\mathbf{a} = 7\hat{\mathbf{i}} - 4\hat{\mathbf{j}} + 7\hat{\mathbf{k}}$
$\mathbf{b} = \hat{\mathbf{i}} - 6\hat{\mathbf{j}} + 10\hat{\mathbf{k}}$
$\mathbf{c} = -\hat{\mathbf{i}} - 3\hat{\mathbf{j}} + 4\hat{\mathbf{k}}$
$\mathbf{d} = 5\hat{\mathbf{i}} - \hat{\mathbf{j}} + \hat{\mathbf{k}}$ respectively.

$\because$ Let point of intersection of $\mathbf{AC}$ and $\mathbf{BD}$ be P



$\therefore$ Position of vector of P is $\left(\frac{\lambda\mathbf{c}+\mathbf{a}}{\lambda+1}\right)$ or
$\frac{\mu\mathbf{d}+\mathbf{b}}{\mu+1}$
$\therefore \frac{-\lambda\hat{\mathbf{i}} - 3\lambda\hat{\mathbf{j}} + 4\lambda\hat{\mathbf{k}} + 7\hat{\mathbf{i}} - 4\hat{\mathbf{j}} + 7\hat{\mathbf{k}}}{\lambda+1}$
$= \frac{5\mu\hat{\mathbf{i}} - \mu\hat{\mathbf{j}} + \mu\hat{\mathbf{k}} + \hat{\mathbf{i}} - 6\hat{\mathbf{j}} + 10\hat{\mathbf{k}}}{\mu+1}$
$\frac{-\lambda+7}{\lambda+1} = \frac{5\mu+1}{\mu+1}$

$$\Rightarrow -\lambda\mu + 7\mu - \lambda + 7 = 5\lambda\mu + \lambda + 5\mu + 1$$
$$6\lambda\mu + 2\lambda - 2\mu = 6$$
$$3\lambda\mu + \lambda - \mu = 3 \qquad \ldots (i)$$
$$\frac{-3\lambda - 4}{\lambda + 1} = \frac{-\mu - 6}{\mu + 1}$$
$$\Rightarrow -3\lambda\mu - 4\mu - 3\lambda - 4 = -\lambda\mu - 6\lambda - \mu - 6$$
$$2\lambda\mu - 3\lambda + 3\mu = 2 \qquad \ldots (ii)$$
$$\frac{4\lambda + 7}{\lambda + 1} = \frac{\mu + 10}{\mu + 1}$$
$$\Rightarrow 4\lambda\mu + 7 + 7\mu + 4\lambda = \lambda\mu + 10\lambda + \mu + 10$$
$$3\lambda\mu - 6\lambda + 6\mu = 3$$
$$\lambda\mu - 2\lambda + 2\mu = 1 \qquad \ldots (iii)$$
$$\lambda(\mu - 2) = 1 - 2\mu$$
$$\lambda = \frac{1 - 2\mu}{\mu - 2}.$$

By Eq. (i)

$$3\mu\left(\frac{1 - 2\mu}{\mu - 2}\right) + \frac{1 - 2\mu}{\mu - 2} - \mu = 3$$
$$\Rightarrow 3\mu - 6\mu^2 + 1 - 2\mu - \mu^2 + 2\mu = 3\mu - 6$$
$$\Rightarrow \mu = \pm 1$$
If $\mu = 1, \quad \lambda = 1$ and $\mu = -1, \quad \lambda = -1$

∴ Position vector of

$$P = \frac{(-\lambda + 7)\hat{\mathbf{i}} + (-3\lambda - 4)\hat{\mathbf{j}} + (4\lambda + 7)\hat{\mathbf{k}}}{\lambda + 1}$$
$$\therefore p + q + r = \frac{-\lambda + 7 - 3\lambda - 4 + 4\lambda + 7}{\lambda + 1}$$
$$= \frac{10}{\lambda + 1} = 5(\text{ At } \lambda = 1)$$

Question32

If $\mathbf{a} = \hat{\mathbf{i}} + \sqrt{11}\hat{\mathbf{j}} - 2\hat{\mathbf{k}}$ and $\mathbf{b} = \hat{\mathbf{i}} + \sqrt{11}\hat{\mathbf{j}} - 10\hat{\mathbf{k}}$ are two vectors, then the component of b perpendicular to a is

Options:

A.

$3\hat{\mathbf{i}} - \sqrt{11}\hat{\mathbf{j}} - 4\hat{\mathbf{k}}$

B.

$\hat{\mathbf{i}} - \sqrt{11}\hat{\mathbf{j}} - 5\hat{\mathbf{k}}$

C.

$-(\hat{\mathbf{i}}+\sqrt{11}\hat{\mathbf{j}}+6\hat{\mathbf{k}})$

D.

$-5\hat{\mathbf{i}}+\sqrt{11}\mathbf{j}+3\hat{\mathbf{k}}$

Answer: C

Solution:

Given, $\mathbf{a}=\hat{\mathbf{i}}+\sqrt{11}\hat{\mathbf{j}}-2\hat{\mathbf{k}}$,

$\mathbf{b}=\hat{\mathbf{i}}+\sqrt{11}\hat{\mathbf{j}}-10\hat{\mathbf{k}}$

$\therefore$ component of $\mathbf{b}$ perpendicular to $\mathbf{a}$ is

$$\mathbf{b}-(\mathbf{a}\cdot\mathbf{b})\hat{\mathbf{a}}$$
$$=(\hat{\mathbf{i}}+\sqrt{11}\hat{\mathbf{j}}-10\hat{\mathbf{k}})$$
$$-\left[\frac{(\hat{\mathbf{i}}+\sqrt{11}\hat{\mathbf{j}}-2\hat{\mathbf{k}})}{\sqrt{16}}\cdot(\hat{\mathbf{i}}+\sqrt{11}\hat{\mathbf{j}}-10\hat{\mathbf{k}})\right]\frac{\hat{\mathbf{i}}+\sqrt{11}\hat{\mathbf{j}}-2\hat{\mathbf{k}}}{\sqrt{16}}$$

$$=\hat{\mathbf{i}}+\sqrt{11}\hat{\mathbf{j}}-10\hat{\mathbf{k}}-\frac{(1+11+20)}{4}\cdot\frac{(\hat{\mathbf{i}}+\sqrt{11}\hat{\mathbf{j}}-2\hat{\mathbf{k}})}{4}$$
$$=\hat{\mathbf{i}}+\sqrt{11\hat{\mathbf{j}}+\sqrt{11}\hat{\mathbf{j}}-2\hat{\mathbf{k}})}$$
$$=-\hat{\mathbf{k}}-2(\hat{\mathbf{i}}+\sqrt{11\hat{\mathbf{k}}}-2\hat{\mathbf{k}})$$
$$=-(\hat{\mathbf{i}}+\sqrt{11}\hat{\mathbf{j}}-6\hat{\mathbf{k}}+6\hat{\mathbf{k}})$$

Question33

Let $\mathbf{a}=\hat{\mathbf{i}}+2\hat{\mathbf{j}}+2\hat{\mathbf{k}}$ and $\mathbf{b}=2\hat{\mathbf{i}}-\hat{\mathbf{j}}+p\hat{\mathbf{k}}$ be two vectors.

If $(\mathbf{a},\mathbf{b})=60^\circ$, then $p=$

Options:

A.

$\frac{\sqrt{7}}{3\sqrt{2}}$

B.

$\frac{3\sqrt{5}}{\sqrt{7}}$

C.

$\frac{\sqrt{3}}{\sqrt{7}}$

D.

$\frac{\sqrt{5}}{\sqrt{7}}$

Answer: B

Solution:

Given, $\mathbf{a}=\hat{\mathbf{i}}+2\hat{\mathbf{j}}+2\hat{\mathbf{k}}$ and $\mathbf{b}=2\hat{\mathbf{i}}-\hat{\mathbf{j}}+p\hat{\mathbf{k}}$

$$\because \mathbf{a}\cdot\mathbf{b}=|\mathbf{a}||\mathbf{b}|\cos\theta$$

$$(2-2+2p)=\sqrt{9}\sqrt{5+p^2}\cdot\cos 60^\circ$$

$$2p=3\sqrt{5+p^2}\frac{1}{2}$$

$$\Rightarrow \quad 4p=3\sqrt{5+p^2}$$

$$\Rightarrow \quad 16p^2=9\left(5+p^2\right)$$

$$\Rightarrow \quad 16p^2=45+9p^2$$

$$\Rightarrow \quad 7p^2=45\Rightarrow p^2=\frac{45}{7}$$

$$\Rightarrow \quad p=\frac{3\sqrt{5}}{\sqrt{7}}$$

Question34

Let π_1 be the plane determined by the vectors $\hat{\mathbf{i}}+\hat{\mathbf{j}}$. $\hat{\mathbf{i}}+\hat{\mathbf{k}}$ and π_2 be the plane determined by the vectors $\hat{\mathbf{j}}-\hat{\mathbf{k}}, \hat{\mathbf{k}}-\hat{\mathbf{i}}$. Let a be a non-zero vector parallel to the line of intersection of the planes π_1 and π_2. If $\mathbf{b}=\hat{\mathbf{i}}+\hat{\mathbf{j}}-\hat{\mathbf{k}}$, then the angle between the vectors a and b is

Options:

A.

$\cos^{-1}\left(\sqrt{\frac{2}{3}}\right)$

B.

$\frac{\pi}{2}$

C.

$\cos^{-1}\left(\frac{1}{\sqrt{3}}\right)$

D.

$\cos^{-1}\left(\frac{\sqrt{2}}{3}\right)$

Answer: A

Solution:

For $\pi_1, n_1 = a \times b$

where, $\mathbf{a} = \hat{\mathbf{i}} + \hat{\mathbf{j}}$

$$\mathbf{b} = \hat{\mathbf{i}} + \hat{\mathbf{k}}$$
$$\therefore \mathbf{n}_1 = \begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{vmatrix} = \hat{\mathbf{i}} - \hat{\mathbf{j}} - \hat{\mathbf{k}}$$

For $\pi_2, \mathbf{n}_2 = \mathbf{c} \times \mathbf{d}$ where $\mathbf{c} = \hat{\mathbf{j}} - \hat{\mathbf{k}}$

$$\mathbf{d} = \hat{\mathbf{k}} - \hat{\mathbf{i}}$$
$$\mathbf{n}_2 = \begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ 0 & 1 & -1 \\ -1 & 0 & 1 \end{vmatrix} = \hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}}$$

Parallel vector line of intersection of π_1 and π_2

$$\mathbf{a} = \mathbf{n}_1 \times \mathbf{n}_2$$
$$\mathbf{n}_2 = \begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ 1 & -1 & -1 \\ 1 & 1 & 1 \end{vmatrix}$$
$$= \hat{\mathbf{i}}(-1+1) - \hat{\mathbf{j}}(1+1) + \hat{\mathbf{k}}(1+1)$$
$$= -2\hat{\mathbf{j}} + 2\hat{\mathbf{k}}$$
$$\text{And } \mathbf{b} = \hat{\mathbf{i}} + \hat{\mathbf{j}} - \hat{\mathbf{k}}$$
$$\therefore \cos\theta = \frac{\mathbf{a} \cdot \mathbf{b}}{|\mathbf{a}||\mathbf{b}|}$$
$$= \left|\frac{-2-2}{\sqrt{4+4}\sqrt{3}}\right| = \left|\frac{4}{2\sqrt{2}\sqrt{3}}\right|$$
$$\theta = \cos^{-1}\sqrt{\frac{2}{3}}.$$

Question35

The variance of the discrete data $3, 4, 5, 6, 7, 8, 10, 13$ is

Options:

A.

7.5

B.

8

C.

9.5

D.

9

Answer: C

Solution:

Data $x_i : 3, 4, 5, 6, 7, 8, 10, 13$

$$\Sigma x_i = 3 + 4 + 5 + 6 + 7 + 8 + 10 + 13 = 56$$
$$\Sigma x_i^2 = 9 + 16 + 25 + 36 + 49 + 64 + 100 + 169$$
$$= 468$$
$$\text{Variance } = \frac{\Sigma x_i^2}{n} - \left(\frac{\Sigma x_i}{n}\right)^2$$
$$= \frac{468}{8} - \left(\frac{56}{8}\right)^2 = 58.5 - 49 = 9.5$$

Question36

If a number x is drawn randomly from the set of numbers $\{1, 2, 3, \ldots, 50\}$, then the probability that number x that is drawn satisfies the inequation $x + \frac{10}{x} \leq 11$ is

Options:

A.

$\frac{4}{5}$

B.

$\frac{9}{50}$

C.

$\frac{4}{25}$

D.

$\frac{1}{5}$

Answer: D

Solution:

$$S = \{1, 2, \ldots, 50\}$$
$$\because x + \frac{10}{x} \leq 11$$
$$x^2 - 11x + 10 \leq 0$$
$$(x - 10)(x - 1) \leq 0$$
$$x \in [1, 10]$$
$$\therefore \text{ Required probability } = \frac{10}{50} = \frac{1}{5}$$

Question37

If a coin is tossed seven times, then the probability of getting exactly three heads such that number two heads occur consecutively is

Options:

A.

$\frac{5}{64}$

B.

$\frac{5}{32}$

C.

$\frac{5}{128}$

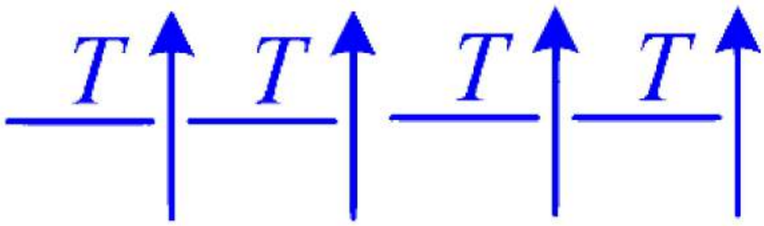
D.

$\frac{35}{128}$

Answer: A

Solution:

7 Toss - 3 Head 4 Tails



$$\therefore \text{ Required probability } = {}^5C_3\left(\frac{1}{2}\right)^7$$
$$= 10 \times \frac{1}{128} = \frac{5}{64}$$

Question38

Two cards are drawn randomly from a pack of 52 playing cards one after the other with replacement. If A is the event of drawing a face card in first draw and B is the event of drawing a clubs card in second draw, then $P\left(\frac{\bar{B}}{A}\right) =$

Options:

A.

$\frac{11}{12}$

B.

$\frac{12}{13}$

C.

$\frac{3}{4}$

D.

$\frac{1}{4}$

Answer: C

Solution:

Step 1: Find Probability of Drawing a Face Card First ($P(A)$)

There are 12 face cards in a deck of 52 cards. So, $P(A) = \frac{12}{52} = \frac{3}{13}$.

Step 2: Find Probability of Drawing a Club Card Second ($P(B)$)

There are 13 clubs in the deck. So, $P(B) = \frac{13}{52} = \frac{1}{4}$.

Step 3: Find $P\left(\frac{\bar{B}}{A}\right)$ (Probability of Not Drawing a Club in Second Draw Given First Card is Face Card)

We use the formula for conditional probability: $P\left(\frac{\bar{B}}{A}\right) = \frac{P(\bar{B}\cap A)}{P(A)}$.

The draws are independent, so $P(\bar{B} \cap A) = P(\bar{B}) \cdot P(A)$.

This means $P\left(\frac{\bar{B}}{A}\right) = \frac{P(\bar{B})\cdot P(A)}{P(A)} = P(\bar{B})$.

We already have $P(B)$, so $P(\bar{B}) = 1 - P(B) = 1 - \frac{1}{4} = \frac{3}{4}$.

Question39

If X is a random variable with probability distribution $P(X = k) = \frac{(2k+3)c}{3^k}, k = 0, 1, 2, \ldots..$ to ∞, then $P(X = 3) =$

Options:

A.

$\frac{1}{24}$

B.

$\frac{1}{18}$

C.

$\frac{1}{6}$

D.

$\frac{1}{3}$

Answer: B

Solution:

$$P(X=k)=\frac{2k+3}{3^k}\cdot C$$

$$\Rightarrow \quad \sum_{K=0}^{\infty} P(X=k)=1$$

$$\Rightarrow \quad \sum_{K=0}^{\infty} \frac{2k+3}{3^k}\cdot c=1$$

$$S=\sum_{K=0}^{\infty} \frac{2k+3}{3^k}$$

$$\Rightarrow \quad S=\frac{3}{3^0}+\frac{5}{3^1}+\frac{7}{3^2}+\frac{9}{3^3}+\ldots$$

$$\Rightarrow \quad \frac{1}{3}S=\frac{3}{3^1}+\frac{5}{3^2}+\frac{7}{3^3}+\ldots$$

$$\Rightarrow \quad \frac{2S}{3}=3+2\left(\frac{1}{3}+\frac{1}{3^2}+\frac{1}{3^3}+\ldots\right)$$

$$=3+2\left(\frac{\frac{1}{3}}{1-\frac{1}{3}}\right)=3+1$$

$$\Rightarrow \quad \frac{2S}{3}=4\Rightarrow S=6$$

$$\therefore \quad 6c=1\Rightarrow c=\frac{1}{6}$$

Now, $P(X=3)=\frac{6+3}{3^3}\times\frac{1}{6}=\frac{1}{18}$.

Question40

If a possion variate X satisfies the relation $P(X=3)=P(X=5)$, then $P(X=4)=$

Options:

A.

$\frac{50}{3e^{\sqrt{20}}}$

B.

$\frac{20000}{3e^{20}}$

C.

$\frac{125}{3e^{10}}$

D.

$\frac{25}{3e^{\sqrt{20}}}$

Answer: A

Solution:

$$\because P(X=K)=\frac{e^{-\lambda}\lambda^K}{K!}$$
$$\because \quad P(X=3)=P(X=5)$$
$$\Rightarrow \quad \frac{e^{-\lambda}\lambda^3}{3!}=\frac{e^{-\lambda}\lambda^5}{5!}$$
$$\Rightarrow \quad \lambda^2=\frac{120}{6}=20$$
$$\Rightarrow \lambda=\sqrt{20}=2\sqrt{5}$$
$$\therefore P(X=4)=\frac{e^{-\lambda}\lambda^4}{4!}=\frac{e^{-2\sqrt{5}},(\sqrt{20})^4}{24}$$
$$=\frac{400}{24e^{2\sqrt{5}}}=\frac{50}{3e^{2\sqrt{5}}}$$

Question41

The equation of the locus of a point, which is at a distance of 5 units from a fixed point $(1,4)$ and also from a fixed line $2x+3y-1=0$ is

Options:

A.

$9x^2+12xy+4y^2-30x-108y+222=0$

B.

$9x^2-12xy+4y^2-30x-98y+220=0$

C.

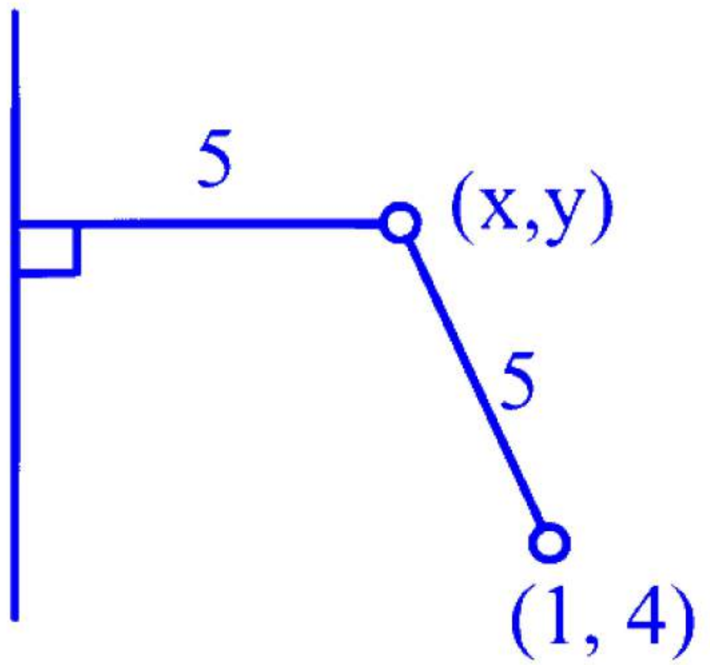
$9x^2+12xy+4y^2-22x-108y+222=0$

D.

$9x^2-12xy+4y^2-22x-98y+220=0$

Answer: D

Solution:



$$\Rightarrow (x-1)^2+(y-4)^2=\frac{(2x+3y-1)^2}{(4+9)}$$
$$\Rightarrow \left(x^2+y^2-2x-8y+1+16\right)(13)=4x^2+9y^2+1+12xy-6y-4x$$
$$\Rightarrow 9x^2+4y^2-12xy-22x-98y+220=0$$

Question42

If $2x^2+xy-6y^2+k=0$ is the transformed equation of $2x^2+xy-6y^2-13x+9y+15=0$ when the origin is shifted to the point (a,b) by translation of axes, then $k=$

Options:

A.

1

B.

0

C.

21

D.

15

Answer: B

Solution:

Now, equation

$2x^2 + xy - 6y^2 + k = 0$

Old equation

$2x^2 + xy - 6y^2 - 13x + 9y + 15 = 0$

Origin is shifted to (a, b)

$2(x-a)^2 + (x-a)(y-b) - 6(y-b)^2 - 13(x-a) + 9(y-b) + 15 = 0$

$\therefore$ Coefficient of $x = 0$

$4a + b - 13 = 0 \qquad \ldots(i)$

Coefficient of $y = 0$

$a - 12b + 9 = 0 \qquad \ldots(ii)$

Solving Eqs. (i) and (ii) we get

$a = 3$ and $b = 1$

$\therefore$ Constant

$\Rightarrow k = 2a^2 + ab - 6b^2 - 13a + 9b + 15$

Put $a = 3$ and $b = 1$

$k = 0$

Question43

The line $L \equiv 6x + 3y + k = 0$ divides the line segment joining the points $(3, 5)$ and $(4, 6)$ in the ratio $-5 : 4$. If the point of intersection of the lines $L = 0$ and $x - y + 1 = 0$ is $P(g, h)$, then $h =$

Options:

A.

$2g$

B.

$2g - 1$

C.

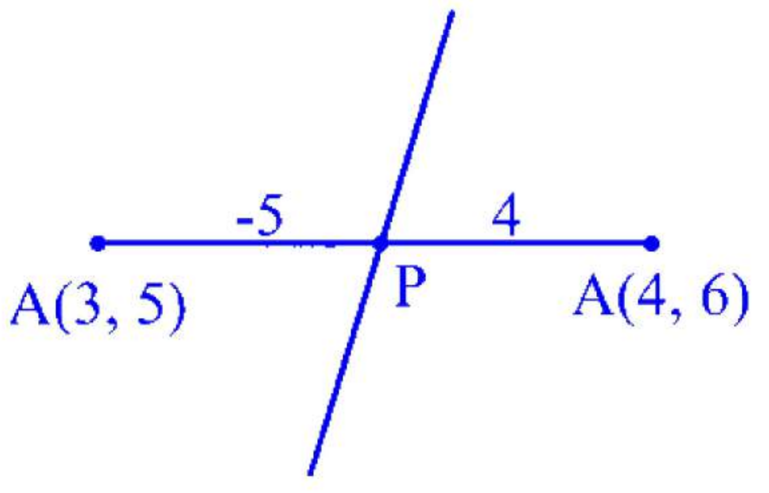
$3g$

D.

$g+1$

Answer: D

Solution:



$$Point\ \ P=\left(\frac{-20+12}{-5+4},\frac{-30+20}{-5+4}\right)$$
$$=\left(\frac{-8}{-1},\frac{-10}{-1}\right)=(8,10)$$

$\because$ P lies on line $6x+3y+k=0$

$$\therefore \quad 6(8)+3(10)+k=0$$
$$k=-78$$

$\therefore$ Equation of line $6x+3y-78=0$

$\Rightarrow 2x+y-26=0$

$\therefore$ Point of intersection of $2x+y-26=0$ and $x-y+1=0$ is $(g,h)=(8,9)$

$$\therefore \quad g-h+1=0$$
$$h=g+1$$

Question44

A straight line through the point $P(1,2)$ makes an angle θ with positive X -axis in anticlockwise direction and meets the line $x+\sqrt{3y}-2\sqrt{3}=0$ at Q. If $PQ=\frac{1}{2}$, then $\theta=$

Options:

A.

$\frac{\pi}{6}$

B.

$\frac{5\pi}{6}$

C.

$\frac{2\pi}{3}$

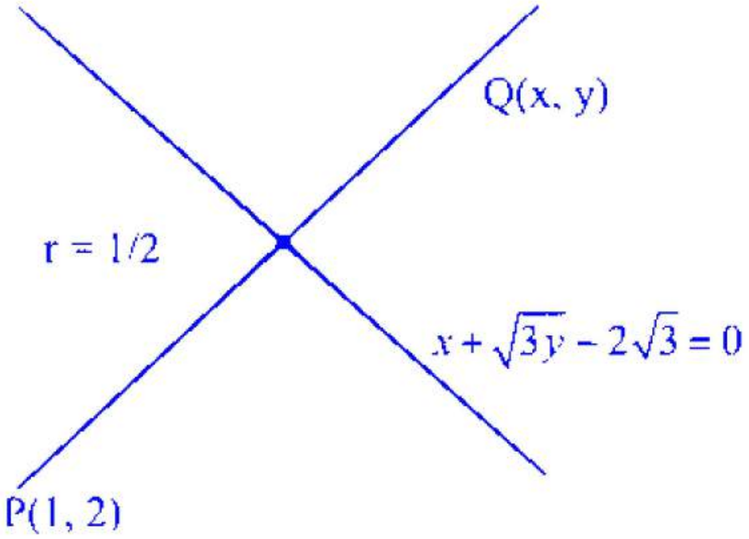
D.

$\frac{\pi}{3}$

Answer: D

Solution:

$$x-1=\pm\frac{1}{2}\cos\theta$$
$$y-2=\pm\frac{1}{2}\sin\theta$$



$x = \pm\dfrac{\cos\theta}{2} + 1$

$y = \pm\dfrac{\sin\theta}{2} + 2$

$\therefore \theta$ lies on line $x + \sqrt{3}y - 2\sqrt{3} = 0$

$\pm\dfrac{\cos\theta}{2} + 1 \pm \dfrac{\sqrt{3}}{2}\sin\theta + 2\sqrt{3} - 2\sqrt{3} = 0$

$\Rightarrow \quad \pm\left(\dfrac{\sqrt{3}}{2}\sin\theta + \dfrac{1}{2}\cos\theta\right) = -1$

$\Rightarrow \quad \sin\left(\theta + \dfrac{\pi}{6}\right) = \pm 1$

$\Rightarrow \quad \theta + \dfrac{\pi}{6} = \dfrac{\pi}{2}$ or $-\dfrac{\pi}{2}$

$\Rightarrow \quad \theta = \dfrac{\pi}{3}$ or $\dfrac{-2\pi}{3}$

Question45

The lines $x - 2y + 1 = 0, 2x - 3y - 1 = 0$ and $3x - y + k = 0$ are concurrent. The angle between the lines $3x - y + k = 0$ and $mx - 3y + 6 = 0$ is 45°. If m is an integer, then $m - k =$

Options:

A.

-6

B.

18

C.

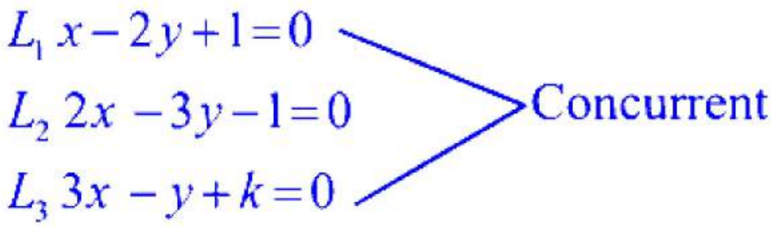
6

D.

-18

Answer: C

Solution:



$$\begin{vmatrix} 1 & -2 & 1 \\ 2 & -3 & -1 \\ 3 & -1 & k \end{vmatrix} = 0$$
$$\Rightarrow 1(-3k-1)+2(2k+3)+1(-2+9)=0$$
$$\Rightarrow -3k-1+4k+6+7=0$$
$$\Rightarrow k=-12$$
$$L_1 : 3x-y+k=0$$
$$\Rightarrow 3x-y-12=0 \Rightarrow m_1=3$$

$$L_2 : mx-3y+6=0$$
$$\Rightarrow \quad m_2=\frac{m}{3}$$
$$\tan\theta=\left|\frac{m_1-m_2}{1+m_1m_2}\right|$$
$$\tan 45^\circ=\left|\frac{3-\frac{m}{3}}{1+m}\right|$$

Question46

If $\tan^{-1}(2\sqrt{10})$ is the angle between the lines $ax^2+4xy-2y^2=0$ and $a \in Z$, then the product of the slopes of given lines is

Options:

A.

$\frac{3}{2}$

B.

$\frac{2}{3}$

C.

$-\frac{2}{3}$

D.

$-\frac{3}{2}$

Answer: D

Solution:

$$
\begin{aligned}
& a x^2+4 x y-2 y^2=0, h=2 a \\
& \tan \theta=\left|\frac{2 \sqrt{h^2-a b}}{a+b}\right| \\
\Rightarrow \quad & 2 \sqrt{10}=\left|\frac{2 \sqrt{4+2 a}}{a-2}\right| \\
\Rightarrow \quad & 10=\frac{4+2 a}{(a-2)^2} \\
\Rightarrow \quad & 5\left(a^2-4 a+4\right)=a+2 \\
\Rightarrow \quad & 5 a^2-21 a+18=0 \\
\Rightarrow \quad & 5 a^2-15 a-6 a+18=0 \\
\Rightarrow \quad & 5 a(a-3)-6(a-3)=0 \\
& a=3, a=\frac{6}{5} \\
\because \quad & a \in Z \\
\therefore \quad & a=3 \\
\therefore \quad & m_1 m_2=\frac{a}{b}=\frac{3}{-2}=\frac{-3}{2} .
\end{aligned}
$$

Question47

If the equation of the circumcircle of the triangle formed by the lines $L_1 \equiv x+y=0$,

$L_2 \equiv 2x+y-1=0, L_3 \equiv x-3y+2=0$ is $\lambda_1 L_1 L_2+\lambda_2 L_2 L_3+\lambda_3 L_3 L_1=0$, then $\frac{7\lambda_1}{\lambda_2}+\frac{\lambda_3}{\lambda_1}=$

Options:

A.

1

B.

2

C.

3

D.

Answer: C

Solution:

$L_1 : x + y = 0$
$L_2 : 2x + y - 1 = 0$
$L_3 : x - 3y + 2 = 0$

Now, equation of circumcircle

$\lambda_1 L_1 L_2 + \lambda_2 L_2 L_3 + \lambda_3 L_3 L_1 = 0$
$\lambda_1(x + y)(2x + y - 1) + \lambda_2(2x + y - 1)$
$(x - 3y + 2) + \lambda_3(x - 3y + 2)(x + y) = 0$

Coefficient of x^2 = Coefficient of y^2

$2\lambda_1 + 2\lambda_2 + \lambda_3 = \lambda_1 - 3\lambda_2 - 3\lambda_3$
$\lambda_1 + 5\lambda_2 + 4\lambda_3 = 0$
$5\lambda_2 = -\lambda_1 - 4\lambda_3 \qquad \ldots(i)$

Coefficient of $xy = 0$

$3\lambda_1 - 5\lambda_2 - 2\lambda_3 = 0$
$5\lambda_2 = 3\lambda_1 - 2\lambda_3 \qquad \ldots(ii)$

From Eqs. (i)and (ii),

$$\therefore \quad -\lambda_1 - 4\lambda_3 = 3\lambda_1 - 2\lambda_3$$
$$4\lambda_1 = -2\lambda_3 \Rightarrow \lambda_3 = -2\lambda_1$$
$$\frac{\lambda_1}{\lambda_3} = \frac{-1}{2}$$

And $5\lambda_2 = -\lambda_1 + 8\lambda_1 = 7\lambda_1$

$\lambda_2 = \frac{7\lambda_1}{5} \Rightarrow \frac{\lambda_1}{\lambda_2} = \frac{5}{7}$
$\therefore \quad \frac{7\lambda_1}{\lambda_2} + \frac{\lambda_3}{\lambda_1} = 7 \times \frac{5}{7} - 2 = 5 - 2 = 3$

Question48

A circle C touches X-axis and makes an intercept of length 2 units on Y-axis. If the centre of this circle lies on the line $y = x + 1$, then a circle passing through the centre of the circle C is

Options:

A.

$x^2 + y^2 - 2x - 4y + 1 = 0$

B.

$x^2 + y^2 - 26x - 20y + 19 = 0$

C.

$x^2 + y^2 - 20x - 26y + 19 = 0$

D.

$x^2 + y^2 + 2x - 4y + 1 = 0$

Answer: B

Solution:

$x_{\text{intercept}} \quad = 0$

$y_{\text{intercept}} \quad = 2$

Centre lies on $y = x + 1$

Let equation of circle

$x^2 + y^2 + 2gx + 2fy + c = 0$

Centre $(-g, -f) \Rightarrow -f = -g + 1$

$$
\begin{aligned}
& f = g - 1 \qquad \qquad \qquad \qquad \ldots (i) \\
& x_{\text{intercept}} \quad = 2\sqrt{g^2 - c} = 0 \\
& \Rightarrow \quad g^2 = c \\
& y_{\text{intercept}} \quad = 2\sqrt{f^2 - c} = 2 \\
& \Rightarrow \quad f^2 - c = 1
\end{aligned}
$$

$$
\begin{aligned}
& f^2 - g^2 = 1 \\
& (f - g)(f + g) = 1 \\
& \quad f + g = -1 \qquad [\because f - g = -1] \\
& \therefore f = -1, \quad g = 0
\end{aligned}
$$

$\therefore$ Centre is $(0, 1)$.

Which satisfies

$x^2 + y^2 - 26x - 20y + 19 = 0$

Question49

If m_1, m_2 are the slopes of the tangents drawn through the point $(-1, -2)$ to the circle $(x-3)^2+(y-4)^2=4$, then $\sqrt{3}\left|m_1-m_2\right|=$

Options:

A.

1

B.

2

C.

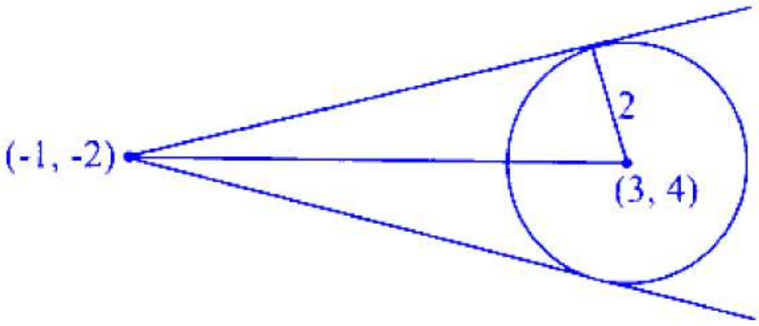
3

D.

4

Answer: D

Solution:



Let equation of tangent be

$y+2=m(x+1) \Rightarrow mx-y+m-2=0$

$\therefore$ 1 st distance from centre = radius

$$\left|\frac{3m-4+m-2}{\sqrt{1+m^2}}\right|=2$$
$$\Rightarrow(4m-6)^2=4\left(1+m^2\right)$$
$$\Rightarrow 3m^2-12m+8=0$$
$$\therefore \sqrt{3}\left|m_1-m_2\right|=\sqrt{3} \frac{\sqrt{D}}{a}$$
$$=\sqrt{3} \frac{\sqrt{144-4 \times 8 \times 3}}{3}$$
$$=\frac{4}{\sqrt{3}} \times \sqrt{3}=4$$

Question50

A line meets the circle $x^2+y^2-4x-4y-8=0$ in two points A and B. If $P(2,-2)$ is a point on the circle such that $PA=PB=2$, then the equation of the line AB is

Options:

A.

$2x+3y=0$

B.

$3x+2y=0$

C.

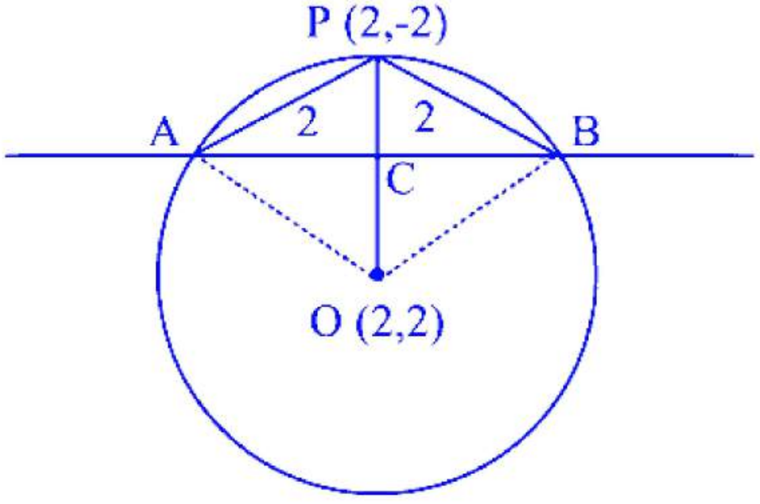
$2x+3=0$

D.

$2y+3=0$

Answer: D

Solution:



$\because \quad PA=PB$
$\therefore \quad \triangle OAP \cong \triangle OPB.$

(O is centre of circle

$x^2 + y^2 - 4x - 4y - 8 = 0$
$r = \sqrt{4+4+8} = 4)$

$\because$ Equation of circle at P and radius 4 is

$(x-2)^2 + (y+2)^2 = 2^2$

$\therefore AB$ is common chord

$\therefore$ Equation is $S_1 - S_2 = 0$

$$\left(x^2 + y^2 - 4x - 4y - 8\right)$$
$$-\left(x^2 + y^2 - 4x + 4y + 4 + 4 - 4\right) = 0$$
$$\Rightarrow \quad -8y - 8 - 4 = 0$$
$$\Rightarrow \quad 8y + 12 = 0$$
$$2y + 3 = 0$$

Question51

If the centre (α, β) of a circle cutting the circles $x^2 + y^2 - 2y - 3 = 0$ and $x^2 + y^2 + 4x + 3 = 0$ orthogonally lies on the line $2x - 3y + 4 = 0$, then $2\alpha + \beta =$

Options:

A.

3

B.

-3

C.

0

D.

1

Answer: B

Solution:

$\because$ If two circles cut orthogonally.

$\Rightarrow 2g_1g_2 + 2f_1f_2 = c_1c_2$
$S_1 : x^2 + y^2 - 2y - 3 = 0$
$S_2 : x^2 + y^2 + 4x + 3 = 0$
$S_3 : x^2 + y^2 - 2\alpha x - 2\beta x + c = 0 \rightarrow$ centre
(α, β)
$\therefore 2(-\alpha)(0) + 2(-\beta)(-1) = c - 3$
$2\beta = c - 3 \quad \ldots (i)$

And $2(-\alpha)(2) + 2\beta(0) = c + 3$

$-4\alpha = c + 3 \Rightarrow 4\alpha = -c - 3 \quad \ldots (ii)$

On adding Eqs. (i) and (ii), we get

$$4\alpha + 2\beta = -6$$
$$\Rightarrow 2\alpha + \beta = -3$$

Question52

The radius of a circle C_1 is thrice the radius of another circle C_2 and the centres of C_1 and C_2 are $(1, 2)$ and $(3, -2)$ respectively. If they cut each other orthogonally and the radius of the circle C_1 is $3r$, then the equation of the circle with r as radius and $(1, -2)$ as centre is

Options:

A.

$x^2 + y^2 - 2x + 4y - 3 = 0$

B.

$x^2 + y^2 - 2x + 4y + 7 = 0$

C.

$x^2 + y^2 - 2x + 4y - 7 = 0$

D.

$x^2 + y^2 - 2x + 4y + 3 = 0$

Answer: D

Solution:

Circle $C_1 \rightarrow$ radius $r_1, O_1(1, 2), r_1 = 3r'$

Circle $C_2 \rightarrow$ radius $r_2, O_2(3,-2), r_2=r'$

$\because C_1$ and C_2 cuts orthogonally

$$r_1^2+r_2^2=0_1 0_2^2$$
$$\Rightarrow \left(3r'\right)^2+r'^2=2^2+4^2$$
$$\Rightarrow 10r'^2=20 \Rightarrow r'=2$$

$\therefore$ Equation of required circle

$$\Rightarrow (x-1)^2+(y+2)^2=2$$
$$\Rightarrow x^2+y^2-2x+4y+3=0$$

Question53

If the normals drawn at the points $P\left(\frac{3}{4}, \frac{3}{2}\right)$ and $Q(3,3)$ on the parabola $y^2=3x$ intersect again on $y^2=3x$ at R, then $R=$

Options:

A.

$(12,6)$

B.

$\left(\frac{27}{4},-\frac{9}{2}\right)$

C.

$\left(\frac{3}{16}, \frac{3}{4}\right)$

D.

$\left(\frac{1}{12},-\frac{1}{2}\right)$

Answer: B

Solution:

Given, $y^2=3x$

Differential w.r.t. x, we get

$2y\frac{dy}{dx}=3 \Rightarrow \frac{dy}{dx}=\frac{3}{2y}$

Slope of tangent at $\left(x_1, y_1\right)=m_t=\frac{3}{2y_1}$

Slope of normal $= m_n = \frac{-2y_1}{3}$

$\therefore$ Equation of normal

$$y - y_1 = \frac{-2y_1}{3}(x - x_1)$$

$$\text{Put } (x_1, y_1) = \left(\frac{3}{4}, \frac{3}{2}\right)$$

$$\Rightarrow \quad y - \frac{3}{2} = -\frac{2\left(\frac{3}{2}\right)}{3} \cdot \left(x - \frac{3}{4}\right)$$

$$\Rightarrow \quad y + x = \frac{3}{4} + \frac{3}{2} = \frac{9}{4}$$

$$\Rightarrow \quad 4x + 4y = 9 \qquad \ldots(i)$$

From $Q(3, 3)$

$$y - 3 = \frac{-2 \times 3}{3}(x - 3)$$

$$\Rightarrow \quad y + 2x = 9 \qquad \ldots(ii)$$

From Eqs. (i) and (ii), we get

$$x = \frac{27}{4}, y = \frac{-9}{2}$$

$$\therefore \quad R \equiv \left(\frac{27}{4}, \frac{-9}{2}\right)$$

Question54

If θ is the acute angle between the tangents drawn from the point $(1, 5)$ to the parabola $y^2 = 9x$, then

Options:

A.

$\frac{\pi}{6} < \theta < \frac{\pi}{4}$

B.

$\frac{\pi}{3} < \theta < \frac{\pi}{2}$

C.

$0 < \theta < \frac{\pi}{6}$

D.

$\frac{\pi}{4} < \theta < \frac{\pi}{3}$

Answer: D

Solution:

$y^2 = 9x$

$\Rightarrow \quad a = \frac{9}{4}$

Equation of tangent to parabola

$$y = mx + \frac{a}{m}$$
$$\Rightarrow \quad y = mx + \frac{9}{4m}$$

Since, tangents are drawn from point (1, 5)

$$\Rightarrow \quad 5 = m + \frac{9}{4m}$$
$$\Rightarrow \quad 4m^2 - 20m + 9 = 0$$
$$\Rightarrow \quad m = \frac{20 \pm \sqrt{400-144}}{8} = \frac{20 \pm 16}{2}$$
$$\Rightarrow \quad m = \frac{9}{2}, \frac{1}{2}$$

$$\therefore \tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right| = \left|\frac{\frac{9}{2} - \frac{1}{2}}{1 + \frac{9}{4}}\right|$$
$$= \frac{16}{13} \approx 1.23$$
$$\Rightarrow \quad 1 < \frac{16}{13} < \sqrt{3}$$
$$\Rightarrow \quad \tan\frac{\pi}{4} < \tan\theta < \tan\frac{\pi}{3}$$
$$\therefore \quad \frac{\pi}{4} < \theta < \frac{\pi}{3}.$$

Question55

Let P be a point on the ellipse $\frac{x^2}{9} + \frac{y^2}{4} = 1$ and let the perpendicular drawn through P to the major axis meet its auxiliary circle at Q. If the normals drawn at P and Q to the ellipse and the auxiliary circle respectively meet in R, then the equation of the locus of R is

Options:

A.

$x^2 + y^2 = 5$

B.

$x^2 + y^2 = 13$

C.

$x^2+y^2=25$

D.

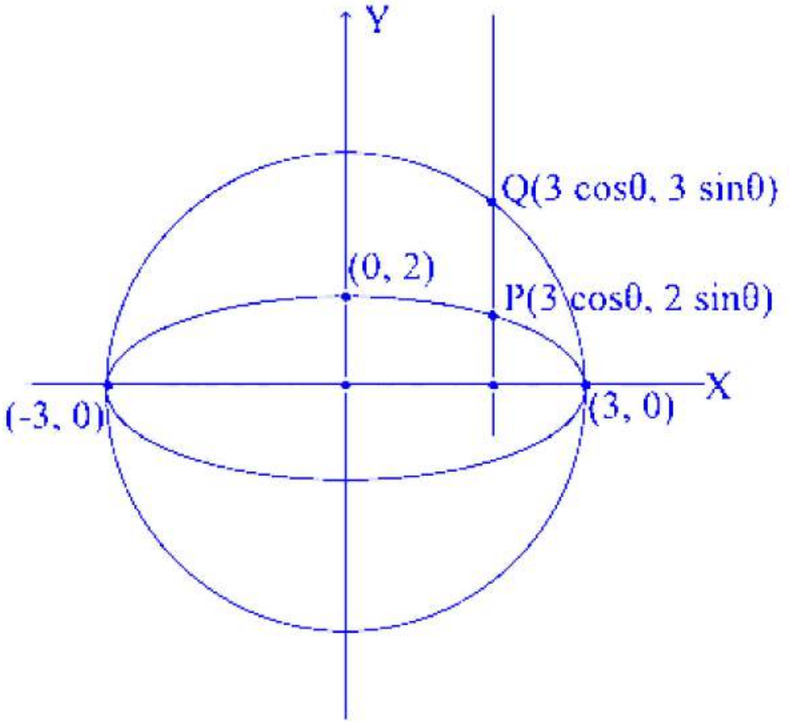
$x^2+y^2=1$

Answer: C

Solution:

We have, $a=3, b=2$

$P(3\cos\theta, 2\sin\theta)$



Equation of normal at P.

$\frac{3x}{\cos\theta}-\frac{2y}{\sin\theta}=5 \qquad \ldots(i)$

Equation of auxiliary circle $x^2+y^2=13$

$\because$ Normal of circle passes through centre

$\therefore Q=(3\cos\theta, 3\sin\theta)$

Normal at Q to circle

$y=x\tan\theta$
$\Rightarrow \quad x=y\cot\theta \qquad \ldots(ii)$

Solving Eqs. (i) and (ii), we get

$y=5\sin\theta$
$x=5\cos\theta$

$\therefore$ Locus of R is $x^2+y^2=25$

Question56

The mid-point of the chord of the ellipse $x^2+\frac{y^2}{4}=1$ formed on the line $y=x+1$ is

Options:

A.

$\left(\frac{4}{5}, \frac{9}{5}\right)$

B.

$\left(-\frac{1}{5}, \frac{4}{5}\right)$

C.

$\left(\frac{1}{5}, \frac{6}{5}\right)$

D.

$\left(-\frac{6}{5},-\frac{1}{5}\right)$

Answer: B

Solution:

Let mid-point of chord be $P\left(x_1, y_1\right)$

$\therefore$ Equation of chord to ellipse $x^2+\frac{y^2}{4}=1$ is $T=S_1$

$$x x_1+\frac{y y_1}{4}-1=x_1^2+\frac{y_1^2}{4}-1$$
$$x x_1+\frac{y y_1}{4}=x_1^2+\frac{y_1^2}{4}$$

Given, $x-y=-1$

$$\frac{x_1}{1}=\frac{\frac{y_1}{4}}{-1}=\frac{x_1^2+\frac{y_1^2}{4}}{-1}$$
$$y_1=-4 x_1 \qquad \ldots(i)$$

And $-x_1=x_1^2+\frac{y_1^2}{4}$

By Eq. (i)

$-x_1 = x_1^2 + 4x_1^2 \Rightarrow 5x_1^2 = -x_1$

$\therefore \quad x_1 = \frac{-1}{5}$ and $y_1 = \frac{4}{5}$

Question57

If the tangent drawn at the point $P(3\sqrt{2}, 4)$ on the hyperbola $\frac{x^2}{9} - \frac{y^2}{16} = 1$ meets its directrix at $Q(\alpha, \beta)$ in fourth quadrant, then $\beta =$

Options:

A.

$\frac{5\sqrt{2}-9}{4}$

B.

$-\frac{9}{5}$

C.

$\frac{12\sqrt{2}-20}{5}$

D.

$-\frac{5}{4}$

Answer: C

Solution:

Tangent at $P(3\sqrt{2}, 4)$ on hyperbola

$$\frac{x^2}{9} - \frac{y^2}{16} = 1 \text{ is } T = 0$$

$$\frac{x(3\sqrt{2})}{9} - \frac{y(4)}{16} = 1$$

$$\Rightarrow \quad \frac{\sqrt{2}x}{3} - \frac{y}{4} = 1$$

$$\Rightarrow \quad 4\sqrt{2}x - 3y - 12 = 0 \qquad \ldots(i)$$

$$\because \quad a = 3, b = 4$$

$$\Rightarrow \quad e = \sqrt{1 + \frac{16}{9}} = \frac{5}{3}$$

Equation of directrix $\Rightarrow x = \pm\frac{a}{e}$.

$$x=\pm\frac{9}{5} \Rightarrow \quad y=\frac{4\sqrt{2}x-12}{3}$$

at $\quad x=\frac{9}{5}$

$$\therefore \quad y=\frac{12\sqrt{2}-20}{5}=\beta$$

Question58

If $m:n$ is the ratio in which the point $\left(\frac{8}{5},-\frac{1}{5},\frac{8}{5}\right)$ divides the segment joining the points $(2,p,2)$ and $(p,-2,p)$, where p is an integer than $\frac{3m+n}{3n}=$

Options:

A.

p

B.

$2p$

C.

$3p$

D.

$4p$

Answer: A

Solution:

Let $\frac{m}{n}=\frac{\lambda}{1}$

$$\frac{p\lambda+2}{\lambda+1}=\frac{8}{5}$$
$$5p\lambda+10=8\lambda+8$$

$$5p\lambda - 8\lambda = -2$$
$$\lambda(8 - 5p) = 2 \quad \ldots (i)$$
$$\frac{-2\lambda + p}{\lambda + 1} = \frac{-1}{5}$$
$$-10\lambda + 5p = -\lambda - 1 \Rightarrow 5p + 1 = 9\lambda$$
$$\lambda = \frac{5p + 1}{9} \quad \ldots (ii)$$
$$\frac{\lambda p + 2}{\lambda + 1} = \frac{8}{5} \quad \ldots (iii)$$

Solving Eqs. (i) and (ii), we get

$$\left(\frac{5p+1}{9}\right)(8 - 5p) = 2$$
$$\Rightarrow \quad (5p + 1)(8 - 5p) = 18$$
$$\Rightarrow \quad 40p - 25p^2 + 8 - 5p - 18 = 0$$
$$\Rightarrow \quad -25p^2 + 35p - 10 = 0$$
$$\Rightarrow \quad 5p^2 - 7p + 2 = 0$$
$$\therefore \quad p = 1 \qquad (\because p \in Z)$$
$$\lambda = \frac{5+1}{9} = \frac{6}{9} = \frac{2}{3}.$$
$$\because \quad \frac{m}{n} = \lambda \therefore \frac{3m+n}{3n} = \frac{m}{n} + \frac{1}{3}$$
$$= \frac{2}{3} + \frac{1}{3} = 1 = p$$

Question59

If $(\alpha, \beta\gamma)$ is the foot of the perpendicular drawn from a point $(-1, 2, -1)$ to the line joining the points $(2, -1, 1)$ and ($1, 1-2$), then $\alpha + \beta + \gamma =$

Options:

A.

2

B.

$-\frac{1}{7}$

C.

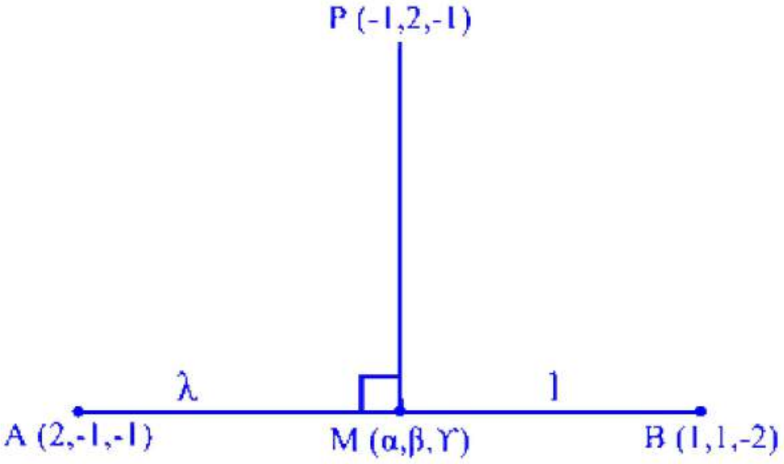
0

D.

$\frac{3}{14}$

Answer: B

Solution:



Let M be the foot of perpendicular

$M=\left(\frac{\lambda+2}{\lambda+1}, \frac{\lambda-1}{\lambda+1}, \frac{-2\lambda+1}{\lambda+1}\right)$

$\because \mathbf{PM} \cdot \mathbf{AB}=0$

$$\left(\frac{\lambda+2}{\lambda+1}+1\right)(1)+\left(\frac{\lambda-1}{\lambda+1}-2\right)(-2)+\left(\frac{-2\lambda+1}{\lambda+1}+1\right)3=0$$

$\therefore 2\lambda+3-2(-\lambda-3)+3(-\lambda+2)=0$

$\Rightarrow 2\lambda+3+2\lambda+6-3\lambda+6=0 \Rightarrow \lambda=-15$

$$\therefore \alpha+\beta+\gamma=\frac{\lambda+2+\lambda-1-2\lambda+1}{\lambda+1}$$

$$=\frac{2}{\lambda+1}=\frac{2}{-15+1}=\frac{-1}{7}$$

Question60

If $A(2,1,-1), B(6,-3,2), C(-3,12,4)$ are the vertices of a $\triangle ABC$ and the equation of the plane containing the $\triangle ABC$ is $53x+by+cz+d=0$, then $\frac{d}{b+c}=$

Options:

A.

-5

B.

1

C.

4

D.

-15

Answer: D

Solution:

Equation of plane containing $A(2,1,-1), B(6,-3,2) C(-3,12,4)$ is

$$\begin{vmatrix} x-2 & y-1 & z+1 \\ 4 & -4 & 3 \\ 9 & -15 & -2 \end{vmatrix} = 0$$
$$\Rightarrow (x-2)(8+45)-(y-1)(-8-27)+(z+1)(-60+36)=0$$
$$\Rightarrow 53x-106+35y-35-24z-24=0$$
$$\Rightarrow 53x+35y-24z-165=0$$
$$\therefore \frac{d}{b+c}=\frac{-165}{35-24}=\frac{-165}{11}=-15$$

Question61

If $\{x\}=x-[x]$, where $[x]$ is the greatest integer $\leq x$ and $\lim_{x\to 0^-}\frac{\cos^{-1}(1-\{x\}^2)\sin^{-1}(1-\{x\})}{\{x\}-\{x\}^4}=\theta$, then $\tan\theta$

Options:

A.

$\frac{1}{\sqrt{3}}$

B.

1

C.

$\sqrt{3}$

D.

∞

Answer: A

Solution:

$$\lim_{x \to 0^-} \frac{\cos^{-1}\left(1-\{x\}^2\right)\sin^{-1}(1-\{x\})}{\{x\}\left(1-\{x\}^3\right)}$$

If $x \to 0^-, \{x\} = 1 + x$

$$\Rightarrow \lim_{x \to 0^-} \frac{\cos^{-1}\left(1-(1+x)^2\right)\sin^{-1}(1-(1+x))}{(1+x)\left(1-(1+x)^3\right)}$$

$$= \lim_{x \to 0^-} \frac{\cos^{-1}\left(-x^2-2x\right)\sin^{-1}(-x)}{(1+x)(-x)\left(x^2+3x+3\right)}$$

$$= \frac{\cos^{-1} 0}{1 \cdot 3} = \frac{\pi}{6}$$

$$\therefore \quad \theta = \frac{\pi}{6} \Rightarrow \tan\theta = \frac{1}{\sqrt{3}}$$

Question62

For $a \neq 0$ and $b \neq 0$, if the real valued function $f(x) = \frac{\sqrt[5]{a(625+x)}-5}{\sqrt[4]{625+bx}-5}$ is continuous at $x = 0$, then $f(0) =$

Options:

A.

$\frac{4b}{5}$

B.

$\frac{5b}{4}$

C.

$\frac{5}{4b}$

D.

$\frac{4}{5b}$

Answer: D

Solution:

$\because f(x)$ is continuous at $x = 0$

$\therefore f(0) = \lim_{x \to 0} \frac{(a(625+x))^{1/5}-5}{(625+bx)^{1/4}-5}$

at $x = 0$

$\frac{(625a)^{1/5}-5}{0}$ (limit exist only when

$(625a)^{1/5} - 5 = 0$
$\therefore \quad a = 5)$

Using L'Hospital rule,

$$f(0) = \lim_{x\to 0} \frac{\frac{1}{5}\left(a(625+x)^{-4/5}\right)\cdot a}{\frac{1}{4}(625+bx)^{-3/4}\cdot b}$$
$$= \frac{\frac{1}{5}(a\cdot 625)^{-4/5}a}{\frac{1}{4}(625)^{-3/4}b} = \frac{4}{5}\frac{\left(5^5\right)^{-4/5}}{\left(5^4\right)^{-3/4}}\cdot\frac{5}{b}$$
$$= \frac{4\cdot 5^{-4}}{5^{-3}\cdot b} = \frac{4}{5b}$$

Question63

If $3^x y^x = x^{3y}$, then the value of $\frac{dy}{dx}$ at $x = 1$ is

Options:

A.

-3

B.

3

C.

$-\frac{1}{3}$

D.

$\frac{1}{3}$

Answer: D

Solution:

$3^x y^x = x^{3y} \qquad \ldots(i)$

At $x = 1 \Rightarrow 3y = 1 \Rightarrow y = \frac{1}{3}$

Taking log both sides in Eq. (i), we get

$\Rightarrow \log 3^x + \log y^x = \log x^{3y}$
$\Rightarrow x \log 3 + x \log y = 3y \log x$

Differential w.r.t. x, we get

$\log 3 + \frac{x}{y}\frac{dy}{dx} + \log y = \frac{3y}{x} + 3\log x \cdot \frac{dy}{dx}$

Put $x = 1, y = \frac{1}{3}$.

$$\log 3 + 3\frac{dy}{dx} + \log\frac{1}{3} = 3 \times \frac{1}{3} + 3\log 1\frac{dy}{dx}$$
$$\Rightarrow \quad 3\frac{dy}{dx} = 1$$
$$\Rightarrow \quad \frac{dy}{dx} = \frac{1}{3}$$

Question64

The value of x at which the real valued function $f(x) = 7|2x+1| - 19|3x-5|$ is not differentiable is

Options:

A.

1,-1

B.

$\frac{1}{2}, -\frac{5}{3}$

C.

$-\frac{1}{2}, \frac{5}{3}$

D.

0,1

Answer: C

Solution:

$$f(x) = 7|2x+1| - 19|3x-5|$$
$$f(x) = \begin{cases} -7(2x+1) + 19(3x-5), & x < -\frac{1}{2} \\ 7(2x+1) + 19(3x-5), & -\frac{1}{2} \le x < \frac{5}{3} \\ 7(2x+1) - 19(3x-5), & x \ge \frac{5}{3} \end{cases}$$

$$f(x)=\begin{cases}43x-102, x\le\frac{-1}{2}\\71x-88, -\frac{1}{2}\le x<\frac{5}{3}\\-43x+102, x\ge\frac{5}{3}\end{cases}$$

$$f'(x)=43\begin{cases}43, x<\frac{-1}{2}\\71, \frac{-1}{2}<x<\frac{5}{3}\\-43, x>\frac{5}{3}\end{cases}$$

$\therefore f(x)$ is differentiable $x=\frac{-1}{2},\frac{5}{3}$

Question65

If $y=\left(1-x^2\right)\tanh^{-1}x$, then $\frac{d^2y}{dx^2}=$

Options:

A.

$\frac{2xy}{\left(1+x^2\right)^2}$

B.

$-\frac{(x+y)}{\left(1-x^2\right)^2}$

C.

$\frac{2(xy)}{1-x^2}$

D.

$-\frac{2(x+y)}{1-x^2}$

Answer: D

Solution:

$$
\begin{aligned}
y &= \left(1-x^2\right)\tanh^{-1} x \\
\frac{dy}{dx} &= \left(1-x^2\right)\cdot\frac{1}{1-x^2} + \tanh^{-1} x(-2x) \\
&= 1 - 2x\tanh^{-1}(x) \\
\frac{d^2y}{dx^2} &= -2\tanh^{-1} x - \frac{2x}{1-x^2} \\
&= -2\frac{\left(1-x^2\right)\tanh^{-1} x + x}{1-x^2} \\
&= -2\frac{(y+x)}{1-x^2}
\end{aligned}
$$

Question66

If $f(x) = \log_{(x^2-2x+1)}\left(x^2-3x+2\right), x \in R - [1,2]$ **and** $x \neq 0$**, then** $f'(3) =$

Options:

A.

1

B.

0

C.

$\log_e 4$

D.

$\log_4 \mathrm{e}$

Answer: D

Solution:

$$
\begin{aligned}
f(x) &= \log_{(x^2-2x+1)} x^2 - 3x + 2 \\
&= \log_{(x-1)^2}(x-2)(x-1) \\
&= \frac{1}{2}\log_{(x-1)}(x-2)(x-1) \\
\Rightarrow f(x) &= \frac{1}{2}\left(1 + \frac{\log(x-2)}{\log(x-1)}\right)
\end{aligned}
$$

$$\Rightarrow f'(x) = \frac{1}{2}\left(\frac{\log(x-1)\cdot\frac{1}{x-2} - \log(x-2)\frac{1}{(x-1)}}{(\log(x-1))^2}\right)$$

$$\therefore f'(3) = \frac{\frac{1}{2}\left(\log 2 - \frac{\log 1}{2}\right)}{(\log 2)^2}$$

$$= \frac{1}{2\log 2} = \frac{1}{\log 4} = \log_4 e$$

Question67

If the normal drawn at the point P on the curve $y^2 = x^3 - x + 1$ makes equal intercepts on the coordinate axes, then the equation of the tangent drawn to the curve at P is

Options:

A.

$x - y = 0$

B.

$x - y = 4$

C.

$x - y = 1$

D.

$x - y = 2$

Answer: A

Solution:

$$y^2 = x^3 - x + 1$$

$$2y\frac{dy}{dx} = 3x^2 - 1$$

$$\frac{dy}{dx} = \frac{3x^2 - 1}{2y}$$

$$m_N = \frac{-1}{\frac{dy}{dx}} = \frac{-2y}{3x^2 - 1}$$

$\because$ Normal makes equal intercepts

$\therefore m_N = \pm 1$

$$\frac{-2y}{3x^2-1} = \pm 1$$
$$3x^2 - 1 = \pm 2y$$
$$9x^4 + 1 - 6x^2 = 4\left(x^3 - x + 1\right)$$
$$9x^4 - 4x^3 - 6x^2 + 4x - 3 = 0$$
$$\therefore \quad x = 1$$
$$y = 1$$

$\therefore$ Tangent at $P(1,1)$

$$y - 1 = 1(x - 1)$$
$$\Rightarrow \quad y = x$$
$$\Rightarrow \quad x - y = 0$$

Question68

If a balloon lying at an altitude of 30 m from an observed at a particular instant is moving horizontally. At the rate of $1 \mathrm{~m} / \mathrm{s}$ away from him, then the rate at which the balloon is moving away directly from the observer at the 40 th second is (in m/s) .

Options:

A.

1.2

B.

0.9

C.

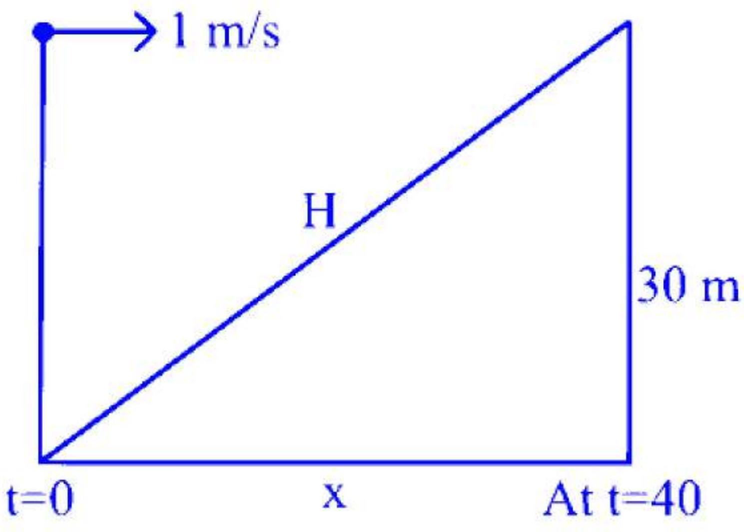
0.6

D.

0.8

Answer: D

Solution:



According to the question,

$x^2 + (30)^2 = H^2$

Differential w.r.t. t, we get

$$2x\frac{dx}{dt} = 2H\frac{dH}{dt}$$
$$\Rightarrow \quad x\frac{dx}{dt} = H\frac{dH}{dt}$$
$$\therefore \quad t = 40 \Rightarrow x = 40$$
$$\therefore \quad H = \sqrt{30^2 + 40^2} = 50$$
$$\therefore \quad 40 \times 1 = 50\frac{dH}{dt}$$
$$\frac{dH}{dt} = \frac{4}{5} = 0.8 \text{ m/s}$$

Question69

The approximate value of $\sqrt{6560}$ is

Options:

A.

80.9939

B.

80.9838

C.

78.9939

D.

78.9838

Answer: A

Solution:

Let $y=\sqrt{x}\frac{dy}{dx}=\frac{1}{2\sqrt{x}}$

$$\left.\frac{dy}{dx}\right|_{x=6561}=\frac{1}{2\sqrt{6561}}=\frac{1}{2\times 81}$$

$x_i=6561$

$x_F=6560, \Delta x=-1$

$\therefore \Delta y=\frac{dy}{dx}, \Delta x.$

$\sqrt{6560}-\sqrt{6561}=\frac{1}{2\times 81}\times -1$

$\therefore \sqrt{6560}=81-\frac{1}{162}$

$=81-0.00617=80.9939$

Question70

A real valued function $f:[4,\infty)\rightarrow R$ is defined as $f(x)=\left(x^2+x+1\right)^{\left(x^2-3x-4\right)}$, then f is

Options:

A.

monotonically decreasing function

B.

monotonically increasing function

C.

increasing in $(4,5)$ and decreasing in $(5,\infty)$

D.

decreasing in $(4,5)$ and increasing in $(5,\infty)$

Answer: B

Solution:

$f(x)=\left(x^2+x+1\right)^{x^2-3x-4}$

Let $g(x)=x^2+x+1, \quad h(x)=x^2-3x-4$

$g'(x)=2x+1, \quad h'(x)=2x-3$
$\therefore \quad x>4$
$g'(x)>0$ and $h'(x)>0$

$\therefore$ both $g(x)$ and $h(x)$ are increasing

$\therefore f(x)$ is monotonically increasing function.

Question71

If a normal is drawn at a variable point $P(x, y)$ on the curve $9x^2+16y^2-144=0$, then the maximum distance from the centre of the curve to the normal is

Options:

A.

1

B.

7

C.

12

D.

$\frac{3}{4}$

Answer: A

Solution:

$$9x^2+16y^2=144$$
$$\Rightarrow \frac{x^2}{16}+\frac{y^2}{9}=1$$

Let point P on ellipse $=P(4\cos\theta, 3\sin\theta)$

Normal at $\boldsymbol{P}$

$$\frac{4x}{\cos\theta} - \frac{3y}{\sin\theta} = 16 - 9$$
$$\Rightarrow \frac{4x}{\cos\theta} - \frac{3y}{\sin\theta} - 7 = 0$$

Distance from origin

$$D = \left| \frac{-7}{\sqrt{\frac{16}{\cos^2\theta} + \frac{9}{\sin^2\theta}}} \right|$$
$$= \left| \frac{-7}{\sqrt{16\sec^2\theta + 9\operatorname{cosec}^2\theta}} \right|$$

for maximum value of D, $P = 16\sec^2\theta + 9\operatorname{cosec}^2\theta$ will be minimum

$\frac{dP}{d\theta} = 32\sec^2\theta\tan\theta - 18\operatorname{cosec}^2\theta\cot\theta = 0$

$$\Rightarrow \quad \frac{\sin\theta}{\cos^3\theta} = \frac{18}{32} \cdot \frac{\cos\theta}{\sin^3\theta}$$
$$\Rightarrow \quad \frac{\sin^4\theta}{\cos^4\theta} = \frac{9}{16}$$
$$\Rightarrow \quad \tan^2\theta = \frac{3}{4} \Rightarrow \cot^2\theta = \frac{4}{3}$$
$$\Rightarrow \quad \sec^2\theta = \frac{7}{4}, \operatorname{cosec}^2\theta = \frac{7}{3}$$
$$D_{\max} = \frac{7}{\sqrt{16 \times \frac{7}{4} + 9 \times \frac{7}{3}}} = \frac{7}{\sqrt{49}} = 1$$

Question72

$$\int e^{-x}\left(x^3 - 2x^2 + 3x - 4\right)dx =$$

Options:

A.

$-e^{-x}\left(x^3 - x^2 + 5x - 1\right) + C$

B.

$e^{-x}\left(x^3 - x^2 + 5x - 1\right) + C$

C.

$e^{-x}\left(x^3 + x^2 + 5x + 1\right) + C$

D.

$-e^{-x}\left(x^3+x^2+5x+1\right)+C$

Answer: D

Solution:

$$I=\int e^{-x}\left(x^3-2x^2+3x-4\right)dx$$

Let $-x=t$

$-dx=dt$

$$\begin{aligned}I&=\int e^t\left(-t^3-2t^2-3t-4\right)(-dt)\\&=\int e^t\left(t^3+2t^2+3t+4\right)dt\\&=\int e^t\left(t^3+3t^2-t^2-2t+5t+5-1\right)dt\\&=\int e^t\left(t^3+3t^2\right)dt+\int e^t\left(-t^2-2t\right)dt+\int e^t(5t+5)dt-\int e^t dt\\&=e^t t^3+e^t\left(-t^2\right)+e^t(5t)-e^t+C\\&=e^t\left(t^3-t^2+5t-1\right)+C\\&=e^{-x}\left(-x^3-x^2-5x-1\right)+C\\&=-e^{-x}\left(x^3+x^2+5x+1\right)+C\end{aligned}$$

Question73

$\int\left(1+\tan^2 x\right)(1+2x\tan x)dx=$

Options:

A.

$x\sec x+C$

B.

$x\tan^2 x+C$

C.

$x\sec^2 x+C$

D.

$x\tan x+C$

Answer: C

Solution:

$$\int \left(1+\tan^2 x\right)(1+2x\tan x)dx$$
$$=\int \left(\sec^2 x+2x\tan x\sec^2 x\right)dx$$
$$=\int \sec^2 x\,dx+\int \underset{I}{2x}\tan \underset{II}{x\sec^2 x}\,dx$$
$$=\tan x+2x\left(\frac{\tan^2 x}{2}\right)-\int \tan^2 x\cdot dx+C$$
$$=\tan x+x\tan^2 x-\int \left(\sec^2 x-1\right)dx+C$$
$$=\tan x+x\tan^2 x-\tan x+x+C$$
$$=x\left(\tan^2 x+1\right)+C=x\sec^2 x+C$$

Question74

$$\int \frac{x^2\tan^{-1}x}{\left(1+x^2\right)^2}dx=$$

Options:

A.

$$\frac{\left(\tan^{-1}x\right)^2}{4}-\frac{x\tan^{-1}x}{2(1+x^2)}+\frac{1-x^2}{4(1+x^2)}+C$$

B.

$$\frac{\left(\tan^{-1}x\right)^2}{4}-\frac{4x\tan^{-1}x+1-x^2}{8(1+x^2)}+C$$

C.

$$\frac{\left(\tan^{-1}x\right)^2}{4}-\frac{x\tan^{-1}x}{(1+x^2)}-\frac{1-x^2}{4(1+x^2)}+C$$

D.

$$\frac{(\tan x)^2}{4}+\frac{4x\tan^{-1}x-1+x^2}{4(1+x^2)}+C$$

Answer: B

Solution:

$$I=\int x^{2}\frac{\tan^{-1}x}{\left(1+x^{2}\right)^{2}}dx$$

Put $x=\tan\theta \Rightarrow dx=\sec^{2}\theta d\theta$

$$I=\int \tan^{2}\theta\cdot\frac{\theta\cdot\sec^{2}\theta d\theta}{\sec^{4}\theta}$$

$$=\int\theta\cdot\frac{\tan^{2}\theta}{\sec^{2}\theta}d\theta=\int\theta\left(\sin^{2}\theta\right)d\theta$$

$$=\int\theta\left(\frac{1-\cos 2\theta}{2}\right)d\theta$$

$$=\frac{1}{2}\int\theta(1-\cos 2\theta)d\theta$$

$$=\frac{1}{2}\int\theta d\theta-\int\theta\cos 2\theta d\theta$$

$$=\frac{1}{2}\left(\frac{\theta^{2}}{2}-\frac{\theta\sin 2\theta}{2}-\frac{\cos 2\theta}{4}\right)+C$$

$$=\frac{\theta^{2}}{4}-\frac{\theta}{4}\sin 2\theta-\frac{1}{8}\cos 2\theta+C$$

$$=\frac{\left(\tan^{-1}x\right)^{2}}{4}-\frac{\tan^{-1}x}{4}\cdot\left(\frac{2x}{1+x^{2}}\right)-\frac{1}{8}\frac{1-x^{2}}{1+x^{2}}+C$$

$$=\frac{\left(\tan^{-1}x\right)^{2}}{4}-\frac{4x\tan^{-1}x+1-x^{2}}{8\left(1+x^{2}\right)}+C$$

Question75

$$\int\frac{\log x}{(1+x)^{3}}dx=$$

Options:

A.

$\frac{1}{2}\left[\frac{1}{1+x}+\frac{\log x}{(1+x)^{2}}-\log\left(x^{2}+x\right)\right]+C$

B.

$\frac{1}{2}\left[\frac{1}{1+x}-\frac{\log x}{(1+x)}-\log\left(1+x^{2}\right)\right]+C$

C.

$\frac{1}{2}\left[\frac{1}{1+x}+\frac{\log x}{(1+x)^{2}}-\log\left(1+x^{2}\right)\right]+C$

D.

$\frac{1}{2}\left[\frac{1}{1+x}-\frac{\log x}{(1+x)^{2}}+\log\left(\frac{x}{1+x}\right)\right]+C$

Answer: D

Solution:

$$
\begin{aligned}
I &= \int \frac{\log x}{(1+x)^3} dx \\
&= \int_1^{\log x} \cdot \frac{1}{(1+x)^3} dx \\
\Rightarrow \quad I &= \log x \int \frac{1}{(1+x)^3} dx - \int \left(\frac{1}{x} \cdot \int \frac{1}{(1+x)^3} dx \right) dx
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{\log x}{-2(1+x)^2} + \frac{1}{2} \int \frac{1}{x} \cdot \frac{1}{(1+x)^2} dx. \\
&= \frac{1}{2} \left(\frac{-\log x}{(1+x)^2} \right) + \frac{1}{2} \int \left(\frac{1}{x} - \frac{1}{1+x} - \frac{1}{(1+x)^2} \right) dx \\
&= \frac{1}{2} \left(\frac{-\log x}{(1+x)^2} + \log x - \log(1+x) + \frac{1}{x+1} \right) + C
\end{aligned}
$$

$= \frac{1}{2} \left(\frac{1}{1+x} - \frac{\log x}{(1+x)^2} + \log \left(\frac{x}{1+x} \right) \right) + C$

Question76

$\int_0^{\pi/4} \frac{1}{5 \cos^2 x + 16 \sin^2 x + 8 \sin x \cos x} dx =$

Options:

A.

$\tan^{-1}\left(\frac{4}{5}\right)$

B.

$2 \tan^{-1}\left(\frac{3}{5}\right)$

C.

$\frac{1}{8} \tan^{-1}\left(\frac{8}{9}\right)$

D.

$\frac{1}{4} \tan^{-1}\left(\frac{7}{8}\right)$

Answer: C

Solution:

$$I=\int_0^{\pi/4}\frac{dx}{5\cos^2 x+16\sin^2 x+8\sin x\cos x}$$
$$=\int_0^{\pi/4}\frac{\sec^2 x dx}{5+16\tan^2 x+8\tan x}$$
Put, $\tan x=t \Rightarrow \sec^2 x dx=dt$
$$I=\int_0^1\frac{dt}{16t^2+8t+5}$$
$$=\frac{1}{16}\int_0^1\frac{dt}{t^2+\frac{1}{2}t+\frac{5}{16}}$$
$$=\frac{1}{16}\int_0^1\frac{dt}{\left(t+\frac{1}{4}\right)^2+\frac{5}{16}-\frac{1}{16}}$$
$$=\frac{1}{16}\int_0^1\frac{dt}{\left(t+\frac{1}{4}\right)^2+\left(\frac{1}{2}\right)^2}$$
$$=\frac{1}{16}\times\frac{1}{\frac{1}{2}}\tan^{-1}\left(\frac{t+\frac{1}{4}}{\frac{1}{2}}\right)_0^1$$
$$=\frac{1}{8}\left[\tan^{-1}\left(\frac{\frac{5}{2}}{2}\right)-\tan^{-1}\left(\frac{1}{2}\right)\right]$$
$$=\frac{1}{8}\tan^{-1}\left(\frac{\frac{5}{2}-\frac{1}{2}}{1+\frac{5}{4}}\right)=\frac{1}{8}\tan^{-1}\left(\frac{8}{9}\right)$$

Question77

$\int_8^{18}\frac{1}{(x+2)\sqrt{x-3}}dx=$

Options:

A.

$\frac{\pi}{6\sqrt{5}}$

B.

$\frac{\pi}{6}$

C.

$\frac{\pi}{3}$

D.

$\frac{\pi}{3\sqrt{5}}$

Answer: A

Solution:

$I=\int_8^{18}\frac{1}{(x+2)\sqrt{x-3}}dx$

$$\begin{aligned}&Put\ \ x-3=t^2\\&\quad dx=2tdt\\&\quad I=\int_{\sqrt{5}}^{\sqrt{15}}\frac{2dt}{(t^2+5)\cdot t}=2\int_{\sqrt{5}}^{\sqrt{15}}\frac{dt}{t^2+5}\\&\quad=\frac{2}{\sqrt{5}}\left(\tan^{-1}\frac{t}{\sqrt{5}}\right)_{\sqrt{5}}^{\sqrt{15}}\\&\quad=\frac{2}{\sqrt{5}}\left(\frac{\pi}{3}-\frac{\pi}{4}\right)=\frac{2}{\sqrt{5}}\left(\frac{\pi}{12}\right)=\frac{\pi}{6\sqrt{5}}\end{aligned}$$

Question78

If [.] denotes the greatest integer function, then $\int_1^2\left[x^2\right]dx=$

Options:

A.

$5+\sqrt{2}+\sqrt{3}$

B.

$5+\sqrt{2}-\sqrt{3}$

C.

$5-\sqrt{2}-\sqrt{3}$

D.

$5-\sqrt{2}+\sqrt{3}$

Answer: C

Solution:

$$
\begin{aligned}
I &= \int_1^2 [x^2]\,dx \\
&= \int_1^{\sqrt{2}} 1dx + \int_{\sqrt{2}}^{\sqrt{3}} 2dx + \int_{\sqrt{3}}^{2} 3dx \\
&= (\sqrt{2}-1) + 2(\sqrt{3}-\sqrt{2}) + 3(2-\sqrt{3}) \\
&= \sqrt{2} - 1 + 2\sqrt{3} - 2\sqrt{2} + 6 - 3\sqrt{3} \\
&= 5 - \sqrt{2} - \sqrt{3}
\end{aligned}
$$

Question79

The differential equation of a family of hyperbolas whose axes are parallel to coordinate axes, centres lie on the line $y = 2x$ and eccentricity is $\sqrt{3}$ is

Options:

A.

$(2x - y)y_2 + y_1^2 - 2y_1 = y_1^3 + 2$

B.

$(y - 2x)y_2 + y_1^2 + 2y_1 = y_1^3 + 2$

C.

$(y - 2x)y_2 - y_1^2 + 2y_1 = y_1^3 - 2$

D.

$(y + 2x)y_2 + y_1^2 + 2y_1 = y_1^3 - 2$

Answer: B

Solution:

Let centre (h, k) lies on $y = 2x$

$$
\begin{aligned}
&\Rightarrow \quad k = 2h \\
&\therefore \quad \frac{(x-h)^2}{a^2} - \frac{(y-2h)^2}{b^2} = 1 \\
&\Rightarrow \quad \frac{(x-h)^2}{a^2} - \frac{(y-2h)^2}{2a^2} = 1 \\
&\qquad \left[\because e = \sqrt{3} \Rightarrow b^2 = 2a^2\right]
\end{aligned}
$$

On differentiating w.r.t. x, we get

$$4(x-h)-2(y-2h)\frac{dy}{dx}=0$$
$$\Rightarrow \quad (y-2h)y_1=2(x-h) \qquad \ldots(i)$$

Again, differentiating,

$(y-2h)y_2=2-y_1^2$

From Eq. (i), $h=\frac{2x-yy_1}{2(1-y_1)}$

∴ Differentiating equation is

$$\left(y-2\left(\frac{2x-yy_1}{2\left(1-y_1\right)}\right)\right)y_2=2-y_1^2$$
$$\Rightarrow \quad (y-yy_1-2x+yy_1)y_2=2+y_1^3-2y_1-y_1^2$$
$$\Rightarrow \quad (y-2x)y_2+y_1^2+2y_1=y_1^3+2$$

Question80

The general solution of the differential equation $\left(x^3-y^3\right)dx=\left(x^2y-xy^2\right)dy$ is

Options:

A.

$y=x\log(c|x+y|)$

B.

$y=\log(c|x+y|)$

C.

$xy=\log(c|x+y|)$

D.

$x+y+\log|x+y|c=0$

Answer: A

Solution:

Given, $\left(x^3-y^3\right)dx=\left(x^2y-xy^2\right)dy$

$$\frac{dy}{dx}=\frac{x^3-y^3}{x^2y-xy^2}$$
$$\frac{dy}{dx}=\frac{x^2+xy+y^2}{xy}$$

Homogeneous differential equation

Put $y=vx$

$$\frac{dy}{dx}=v+x\frac{dv}{dx}$$
$$\Rightarrow v+\frac{xdv}{dx}=\frac{1+v+v^2}{v}$$
$$\Rightarrow x\frac{dv}{dx}=\frac{1+v}{v}$$
$$\Rightarrow \frac{v}{v+1}dv=\frac{dx}{x}$$
$$\Rightarrow \frac{v+1-1}{v+1}dv=\frac{dx}{x}$$

On taking integration both sides, we get

$$\int\frac{v+1}{v+1}dv-\int\frac{1}{v+1}dv=\int\frac{dx}{x}$$
$$v-\ln|v+1|=\ln x+\ln c$$
$$\Rightarrow v=\ln|x(v+1)c|$$
$$\Rightarrow \frac{y}{x}=\log[(y+x)c]$$
$$\Rightarrow y=x\log(c|x+y|)$$

Chemistry

Question1

The energy associated with electron in first orbit of hydrogen atom is -2.18×10^{-18} J. The frequency of the light required (in Hz) to excite the electron to fifth orbit is ($h=6.6\times10^{-34}$Js)

Options:

A.

2.17×10^{16}

B.

3.17×10^{14}

C.

2.17×10^{15}

D.

3.17×10^{15}

Answer: D

Solution:

Energy in the nth orbit is given by

$E_n = \frac{E_1}{n^2}$

For $n = 5$,

$$\begin{aligned} E_5 &= \frac{-2.18 \times 10^{-18} \text{ J}}{(S^2} \\ &= -8.72 \times 10^{-20} \text{ J} \\ \Delta E &= E_5 - E_1 \\ &= \left(-8.72 \times 10^{-20}\right) - \left(-2.18 \times 10^{-18}\right) \\ &= 2.0928 \times 10^{-18} \text{ J} \\ v &= \frac{\Delta E}{n} = \frac{2.0928 \times 10^{-18}}{6.6 \times 10^{-34}} \\ &= 3.17 \times 10^{15} \text{ Hz} \end{aligned}$$

Question2

In $\mathrm{Sr}(Z = 38)$, the number of electrons with $l = 0$ is x, number of electrons with $l = 2$ is $y \cdot (x - y)$ is equal to ($l =$ Azimuthal quantum number)

Options:

A.

0

B.

8

C.

-2

D.

Answer: A

Solution:

Step 1: Write the electron configuration of Strontium (Sr, $Z = 38$)

$Sr(Z = 38) = 1s^2\ 2s^2\ 2p^6\ 3s^2\ 3p^6\ 4s^2\ 3d^{10}\ 4p^6\ 5s^2$

Step 2: Count electrons with $l = 0$

Electrons with $l = 0$ are in s orbitals. Add up all the electrons in s orbitals:
$x = 2(1s) + 2(2s) + 2(3s) + 2(4s) + 2(5s) = 10$ So, $x = 10$

Step 3: Count electrons with $l = 2$

Electrons with $l = 2$ are in d orbitals. Look for the d orbital electrons: $y = 10$ (from $3d^{10}$) So, $y = 10$

Step 4: Find $x - y$

Subtract to get: $x - y = 10 - 10 = 0$

Step 5: Find $y \cdot (x - y)$

Multiply y and $x - y$: $y \cdot (x - y) = 10 \times 0 = 0$

Question3

Match the following.

List-I (Element)		List-II $(\Delta_{eg}H)$ (in $kJmol^{-1}$)	
A.	O	I.	-200
B.	F	II.	-349
C.	Cl	III.	-141
D.	S	IV.	-328
		V.	+48

The correct answer is

Options:

A.

A-II, B-IV, C-I, D-III

B.

A-V, B-IV, C-II, D-I

C.

A-III, B-IV, C-II, D-I

D.

A-III, B-II, C-IV, D-I

Answer: C

Solution:

Let's determine the electron gain enthalpies for the given elements based on periodic trends and known values. Electron gain enthalpy ($\Delta_{eg}H$) is the energy change when an electron is added to a neutral gaseous atom. A more negative value indicates a greater tendency to accept an electron (more exothermic process).

General Trends:

1. **Group 17 (Halogens) have the most negative electron gain enthalpies.** Among halogens, chlorine (Cl) generally has a more negative electron gain enthalpy than fluorine (F) because F's small size leads to significant interelectronic repulsion when an incoming electron is added, making it less favorable than Cl.
    - **Chlorine (Cl):** Expect a very large negative value.
    - **Fluorine (F):** Expect a large negative value, but slightly less negative than Cl.
2. **Group 16 (Chalcogens) also have negative electron gain enthalpies.** Similar to halogens, sulfur (S) generally has a more negative electron gain enthalpy than oxygen (O) due to O's small size and resulting interelectronic repulsion.
    - **Sulfur (S):** Expect a negative value.
    - **Oxygen (O):** Expect a negative value, but less negative than S.

Let's examine the provided values in List-II:

I. -200 $kJmol^{-1}$

II. -349 $kJmol^{-1}$

III. -141 $kJmol^{-1}$

IV. -328 $kJmol^{-1}$

V. +48 $kJmol^{-1}$ (This is a positive value, meaning energy is required to add an electron, which is characteristic of elements like noble gases or alkaline earth metals, but not O, F, Cl, S for the first electron gain).

Now let's match the elements:

1. **Chlorine (Cl):** It is known to have the most negative electron gain enthalpy among all elements. Among the given negative values, -349 $kJmol^{-1}$ is the most negative.
    - So, **C (Cl) $\rightarrow$ II (-349 $kJmol^{-1}$)**.

2. **Fluorine (F):** It has a very large negative electron gain enthalpy, but due to its smaller size and interelectronic repulsion, it's slightly less exothermic than Cl. The second most negative value is -328 $kJmol^{-1}$.
   - So, **B (F) → IV (-328 $kJmol^{-1}$)**.
3. **Sulfur (S):** Belonging to Group 16, its electron gain enthalpy is exothermic, and it should be more negative than Oxygen. Comparing the remaining negative values (-200 and -141), -200 $kJmol^{-1}$ is more negative.
   - So, **D (S) → I (-200 $kJmol^{-1}$)**.
4. **Oxygen (O):** Also in Group 16, its small size leads to interelectronic repulsion, making its electron gain enthalpy less negative (less exothermic) than that of Sulfur. The remaining negative value is -141 $kJmol^{-1}$.
   - So, **A (O) → III (-141 $kJmol^{-1}$)**.

Let's summarize the matching:

- A (O) → III (-141 $kJmol^{-1}$)
- B (F) → IV (-328 $kJmol^{-1}$)
- C (Cl) → II (-349 $kJmol^{-1}$)
- D (S) → I (-200 $kJmol^{-1}$)

Comparing this with the given options:

Option C: A-III, B-IV, C-II, D-I

This matches our derived assignments.

The final answer is $\boxed{\text{Option C}}$

Question4

Observe the following data ($\Delta_t H_1, \Delta_t H_2$ and $\Delta_{eg} H$ represent the first, second ionisation enthalpies and electron gain enthalpy respectively)

Element	$\Delta_l H_1$ $(kJmol^{-1})$	$\Delta_1 H_2$ $(kJmol^{-1})$	$\Delta_{eg} H$ $(kJmol^{-1})$
I	520	7300	-60
II	490	3051	-48
III	1681	3374	-328
IV	2372	5251	+48

Using the data identify the most reactive metal.

Options:

A.

II

B.

I

C.

IV

D.

III

Answer: A

Solution:

Among the given elements, element II is most reactive metals.

The reason is that, it has low 1st ionisation enthalpy, high 2nd ionisation enthalpy and low electron gain enthalpy.

Question5

The sum of bond order of O_2^+, O_2^-, O_2 and O_2^{2+} is equal to

Options:

A.

5

B.

4

C.

6

D.

9

Answer: D

Solution:

Bond order

$= \frac{\text{Bonding electrons - Non-bonding electrons}}{2}$

B.O. of $O_2 = \frac{1}{2}[10-6] = 2$

$O_2^+ = \frac{1}{2}[10-5] = 2.5$

$O_2^- = \frac{1}{2}(10-7) = 1.5$

$O_2^{2+} = \frac{1}{2}(10-4) = 3$

Sum of B.O. of O_2, O_2^+, O_2^- and O_2^{2+}

$= 2 + 2.5 + 1.5 + 3 = 9$

Question6

Observe the following statements

Statement-I Hybridisation is not same in both SF_6 and BrF_5.

Statement-II BrF_5 is square pyramidal while SF_6 is octahedral in shape.

The correct answer is

Options:

A.

Both statement I and II are correct.

B.

Statement I is correct, but statement II is not correct.

C.

Statement I is not correct, but statement II is correct.

D.

Both statement I and II are not correct.

Answer: C

Solution:

Statement I is not correct but statement II is correct. The correct form of statement I is,

Both SF_6 and BrF_5 has same hybridisation i.e., sp^3d^2.

Question7

At $T(\mathrm{~K})$ root mean square (rms) velocity of argon (molar mass $40 \mathrm{~g} \mathrm{~mol}^{-1}$) is $20 \mathrm{~ms}^{-1}$. The average kinetic energy of the same gas at $T(\mathrm{~K})$ (in Jmol^{-1}) is

Options:

A.

8

B.

16

C.

4

D.

2

Answer: A

Solution:

The root mean square (rms) speed of argon gas is given as $20 \mathrm{~ms}^{-1}$.

The molar mass of argon is $40 \mathrm{~g} \mathrm{~mol}^{-1}$. This can also be written as $0.040 \mathrm{~kg} \mathrm{~mol}^{-1}$ (because $1000 \mathrm{~g}=1 \mathrm{~kg}$).

The formula for average kinetic energy per mole of a gas is: $E=\frac{1}{2} M v_{\mathrm{rms}}^2$, where M is the molar mass in kg/mol and v_{rms} is the rms speed.

Substitute the values:
$E = \frac{1}{2} \times 0.040 \times (20)^2$

Calculate $(20)^2 = 400$, so:

$E = \frac{1}{2} \times 0.040 \times 400$

$\frac{1}{2} \times 0.040 = 0.020$, so:
$E = 0.020 \times 400 = 8$

So, the average kinetic energy per mole of argon is 8 J mol^{-1} at this temperature.

Question8

4.0 g of a mixture containing $\mathrm{Na_2CO_3}$ and $\mathrm{NaHCO_3}$ is heated to 673 K . Loss in mass of the mixture is found to be 0.62 g . The percentage of sodium carbonate in the mixture is

Options:

A.

42

B.

58

C.

48

D.

52

Answer: B

Solution:

The decomposition reaction is,

$2\mathrm{NaHCO_3} \xrightarrow{\text{Heat}} \mathrm{Na_2CO_3} + \mathrm{CO_2} + \mathrm{H_2O}$.

Total mass of mixture $= 4$ g

Mass loss $= 0.62$ g

From reaction

$168\ \mathrm{g NaHCO_3} \longrightarrow 62\ \mathrm{g\ loss}$
$1\ \mathrm{g NaHCO_3} \longrightarrow 0.369\ \mathrm{g\ loss}$

Let $x =$ mass of $NaHCO_3$

$x = \frac{168}{62} \times 0.62 = 1.68\ \mathrm{g}$

Mass of $NaCO_3 = 4.0 - 1.68 = 2.32\ \mathrm{g}$

$\% Na_2CO_3 = \frac{2.32}{4.0} \times 100 = 58\%$

Question9

At 298 K , if the standard Gibbs energy change $\Delta_r G^\circ$ of a reaction is -115 kJ , the value of $\log_{10} K_p$ will be $\left(R = 8.314 \mathrm{JK}^{-1}\ \mathrm{mol}^{-1}\right)$

Options:

A.

+20.15

B.

-20.15

C.

-10.30

D.

+10.30

Answer: A

Solution:

Gibbs' free energy is given by

$$\begin{aligned} \Delta G^\circ &= -RT \ln(K) \\ \Rightarrow \quad -115000\ \mathrm{J} &= -8.314 \times 298 \times \ln K_p \\ \Rightarrow \quad \log_{10}(K_p) &= \frac{46.42}{2.303} \\ &\log_{10}(K_p) = +20.15 \end{aligned}$$

Question10

200 mL of an aqueous solution of $\mathrm{HCl}(\mathrm{pH}=2)$ is mixed with 300 mL of aqueous solution of NaOH $(\mathrm{pH}=12)$ and is diluted to 1.0 L . The pH of the resulting solution is ($\mathrm{pH}=2$)

Options:

A.

10.3

B.

11.0

C.

11.3

D.

11.7

Answer: B

Solution:

For HCl, $[\mathrm{H}^{+}]=10^{-\mathrm{pH}}$,

$\mathrm{HCl}=10^{-2}\mathrm{M}$

For NaOH ,

$[\mathrm{OH}^{-}]=10^{-\mathrm{pOH}}, \mathrm{NaOH}=10^{-2}\mathrm{M}$

$$\begin{aligned} \text{Moles of } \mathrm{H}^{+} &= [\mathrm{H}^{+}] \times V_{\mathrm{HCl}} = 10^{-2} \times 0.2 \\ &= 0.002 \text{ mol} \\ \text{Moles of } \mathrm{OH}^{-} &= [\mathrm{OH}^{-}] \times V_{\mathrm{NaOH}} \\ &= 10^{-2} \times 0.300 = 0.003 \text{ mol} \end{aligned}$$

Moles of OH^{-}remaining

$= \text{Moles of OH}^- - \text{Moles of H}^+$
$= 0.003 - 0.002 = 0.001 \text{ mol}$
$[\text{OH}^-]_{\text{final}} = \dfrac{\text{Moles of OH}^-\text{remaining}}{V_{\text{final}}}$
$= \dfrac{0.001}{1} = 0.001\text{M}$
$\text{pOH} = -\log[\text{OH}^-]_{\text{final}} = -\log(0.001) = 3$
$\text{pH} = 14 - \text{pOH} = 14 - 3 = 11$

Question11

Identify the electron rich hydrides from the following

Options:

A.

B_2H_6, AlH_3

B.

NaH, MgH_2

C.

HCl, H_2S

D.

CH_4, SiH_4

Answer: C

Solution:

Among the given options, HCl and H_2 S are electron rich hydrides. Electron rich hydrides are those compounds where the central atom has more electron than required to form single covalent bonds with its surrounding atoms.

Question12

The incorrect statement about Castner-kellner cell process is

Options:

A.

sodium hydroxide is prepared.

B.

brine solution is the electrolyte.

C.

mercury acts as anode and carbon rod acts as cathode.

D.

chlorine gas liberates at anode.

Answer: C

Solution:

Statement given in option (c) is incorrect about Castner-Kellner cell process. It's correct form is, mercury acts as cathode and carbon rod acts as anode.

Question13

The incorrect statement about Castner-kellner cell process is

Options:

A.

sodium hydroxide is prepared.

B.

brine solution is the electrolyte.

C.

mercury acts as anode and carbon rod acts as cathode.

D.

chlorine gas liberates at anode.

Answer: C

Solution:

The Castner-Kellner cell process is an electrolytic method for the industrial production of sodium hydroxide, chlorine, and hydrogen from brine. Let's evaluate each statement:

- **Option A: sodium hydroxide is prepared.**

  In the Castner-Kellner cell, sodium ions are reduced at the mercury cathode to form sodium amalgam. This amalgam is then reacted with water in a separate decomposer unit to produce sodium hydroxide ($NaOH$) and hydrogen gas. So, this statement is **correct**.

- **Option B: brine solution is the electrolyte.**

  The cell uses an aqueous solution of sodium chloride ($NaCl$), which is commonly known as brine, as the electrolyte. So, this statement is **correct**.

- **Option C: mercury acts as anode and carbon rod acts as cathode.**

  In the Castner-Kellner cell:

  - **Anode:** Graphite (carbon) rods are used as the anode, where chloride ions are oxidized to chlorine gas ($2Cl^-(aq) \rightarrow Cl_2(g) + 2e^-$).
  - **Cathode:** A flowing layer of mercury acts as the cathode, where sodium ions are reduced to form sodium amalgam ($Na^+(aq) + e^- \rightarrow Na(\text{in Hg})$).

  Therefore, the statement that mercury acts as the anode and carbon rod acts as the cathode is **incorrect**. The roles are reversed.

- **Option D: chlorine gas liberates at anode.**

  As mentioned above, at the graphite anode, chloride ions are oxidized to produce chlorine gas. So, this statement is **correct**.

The incorrect statement is C.

The final answer is $\boxed{C}$

Question14

Which of the following is an incorrect statement about the compounds of group 13 elements?

Options:

A.

All the trihalides exist except TlI_3.

B.

Trihalides on hydrolysis form tetrahedral species.

C.

Diborane is an example of electron precise hydride.

D.

Hydrolysis of diborane gives boric acid.

Answer: C

Solution:

Among the given statements, statement given in option (c) is incorrect about compounds of group 13 elements. It's correct form is, Diborane (B_2H_6) is an example of electron deficient hydride.

Question15

The incorrect statement about the oxidation states of group 14 elements is

Options:

A.

In addition to $+4, +2$ carbon also shows negative oxidation states.

B.

Tin in +2 state acts as a reducing agent.

C.

Lead in +2 state acts as good reducing agent.

D.

Lead in +4 state acts as a good oxidising agent.

Answer: C

Solution:

Among the given statements, statement given in option (c) is incorrect about oxidation state of group 14 elements, it's correct form is lead in +2 state is not a good reducing agent. This is because it readily accepts two more electrons to achieve the stable +4 oxidation state.

Question16

In drinking water, if the maximum prescribed concentration of copper is $x \mathrm{mgdm}^{-3}$, the maximum prescribed concentration of zinc will be

Options:

A.

$\frac{15}{x}$

B.

$\frac{x}{15}$

C.

$\frac{6}{10} x$

D.

$\frac{5}{6} x$

Answer: A

Solution:

The maximum prescribed concentration of zinc in drinking water is $15 \mathrm{mgdm}^{-3}$. The maximum prescribed concentrations of copper is $1 \mathrm{mgdm}^{-3}$ Thus, the correct option is $\frac{15}{x}$.

Question17

The empirical formula of the compound ' D ' formed in the given reaction sequence is

$$C_2H_4 \xrightarrow{Br_2/CCl_4} A \xrightarrow[\text{(ii) NaNH}_2]{\text{(i) Alc·KOH}} B \xrightarrow[\text{polymerisation}]{\text{Cyclic}} C \xrightarrow[\text{DryAlCla}\quad {}_3\text{, dark.cold}]{Cl_2\text{ (excess)}}$$

Options:

A.

CHCl

B.

CCl

C.

CH_2Cl

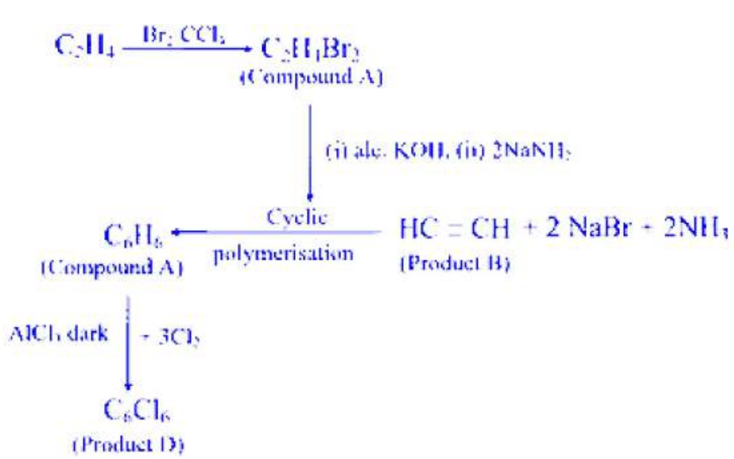
D.

$CHCl_2$

Answer: B

Solution:

The complete reaction mechanism is, as follows,



C_6Cl_6 is the molecular formula of product D. Thus, its, empirical formula is CCl.

Question18

Which one of the following mixtures can be separated by steam distillation technique?

Options:

A.

n-Hexane $+ n$-Heptane

B.

$CHCl_3$ + Aniline

C.

Aniline $+ H_2O$

D.

Glucose +NaCl

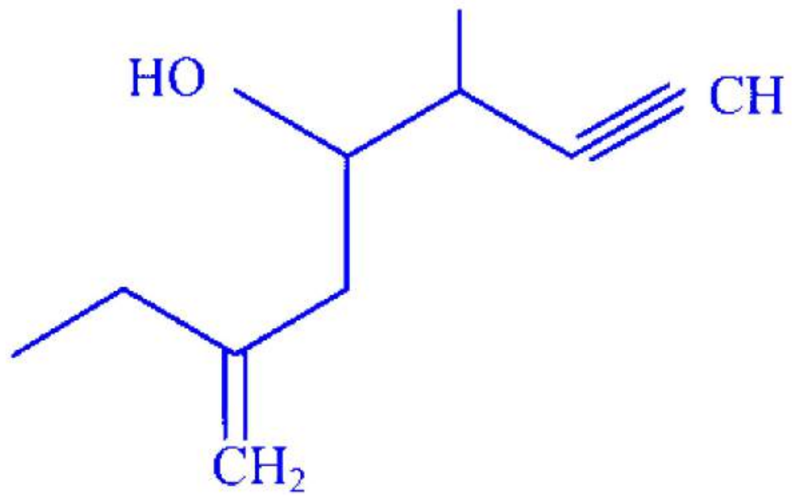
Answer: C

Solution:

Aniline and H_2O can be separated by using steam distillation. They have different boiling point, thus can be seperated by using vapour formation.

Question19

The IUPAC name of the following compound is



Options:

A.

3-methenyl-6-methyloct -7-yn-5-ol

B.

2-ethyl-5-methylhept -1-en-6-yn-4-ol

C.

2-ethyl-5-methylhept-1-yn-6-en-4-ol

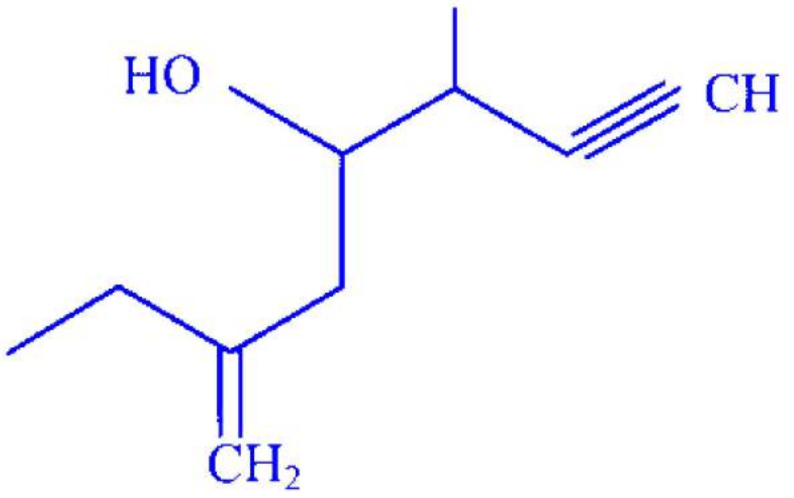
D.

3-methyl-6-ethylhept-6-en-1-yn-4-ol

Answer: B

Solution:

The correct IUPAC name of



2-ethyl-5-methylhept-1-en-6-yn-4-ol

Question20

An alkyne has the molecular formula C_6H_{10}. The number of 1 -alkyne isomers (excluding stereoisomers) possible for it is

Options:

A.

2

B.

5

C.

3

D.

4

Answer: D

Solution:

The number of 1-alkyne isomer possible for the compound with the molecular formula C_6H_{10} is 4 . This are as follows,

- $CH \equiv C - CH_2 - CH_2 - CH_2 - CH_3$

(1-hexyne)

- $CH \equiv C - CH(CH_3) - CH_2 - CH_3$

(3-methyl-1-pentyne)

- $CH \equiv C - CH_2 - CH(CH_3) - CH_3$

(4-methyl-1-pentyne)

- $CH \equiv C - C(CH_3)_2 - CH_3$

(3,3-dimethyl-1-butyne)

Thus, 4 isomers are possible.

Question21

A metal crystallises in two cubic phases, fcc and bcc with edge lengths $3.5\overset{o}{A}$ and $3\overset{o}{A}$ respectively. The ratio of densities of fcc and bcc is approximately

Options:

A.

1.36

B.

1.26

C.

2.16

D.

6.13

Answer: B

Solution:

Density $(d) = \frac{Z \times M}{N_A \times a^3}$

Ratio of density of fcc and bcc,

$$\frac{d_{\mathrm{fcc}}}{d_{\mathrm{bcc}}}=\frac{Z_{fcc}\times a_{bcc}^3}{Z_{bcc}\times a_{fcc}^3}$$

$$\Rightarrow\quad \frac{d_{fcc}}{d_{bcc}}=\frac{4}{2}\times\left(\frac{3}{3.5}\right)^3$$

$$\approx 1.26$$

Question22

Observe the following data given in the table (K_H = Henry's law constant)

Gas	CO_2	Ar	HCHO	CH_4
($\boldsymbol{K}_{\mathrm{H}}$ bar at **298 K**)	1.67	40.3	1.83×10^{-5}	0.413

The correct order of their solubility in water is

Options:

A.

$CO_2 > CH_4 > HCHO > Ar$

B.

$Ar > HCHO > CH_4 > CO_2$

C.

$HCHO > CH_4 > CO_2 > Ar$

D.

$CO_2 > HCHO > CH_4 > Ar$

Answer: C

Solution:

Using Henry's law,

Solubility of gas in liquid $\propto \frac{1}{K_{\mathrm{H}}}$

Lower the value of K_{H}, higher will be the solubility.

Thus, the correct order of solubility is

$HCHO > CH_4 > CO_2 > Ar$

Question23

The Gibbs energy change of the reaction (in $\mathrm{kJmol^{-1}}$) corresponding to the following cell

$\mathrm{Cr} \;\; \mathrm{Cr}^{3+}(0.1\mathrm{M}) \| \mathrm{Fe}^{2+}(0.001\mathrm{M}) \;\; \mathrm{Fe}$

(Given $E^{\circ}_{\mathrm{Cr}^{3+}|\mathrm{Cr}} = -0.75\ \mathrm{V}; E^{\circ}_{\mathrm{Fe}^{2+}|\mathrm{Fe}} = -0.45\ \mathrm{V}$,

$\mathrm{IF} = 96,500\mathrm{Cmol}^{-1})$

Options:

A.

-150.9

B.

-173.7

C.

+150.9

D.

+173.7

Answer: A

Solution:

$$\begin{aligned} E^{\circ}_{\text{cell}} &= E^{\circ}_{\text{cathode}} - E^{\circ}_{\text{anode}} \\ &= (-0.45) + (0.75) = 0.30\ \mathrm{V} \end{aligned}$$

For the reaction:

$$2Cr + 3Fe^{2+} \longrightarrow 2Cr^{3+} + 3Fe$$

$$Q = \frac{[Cr^{3+}]^2}{[Fe^{2+}]^3} = \frac{(0.1)^2}{(0.001)^2} = 10^4$$

$$E_{cell} = 0.30 - \frac{0.059}{6}\log 10^4$$

$$= 0.030 - \frac{0.059}{6} \times 4$$

$$\approx 0.2607 \text{ V}$$

Gibb's free energy,

$$\begin{aligned} \Delta G^\circ &= -nFE^\circ \\ &= 6 \times 96500 \times 0.2607 \times 10^{-3} \\ &= -150.9 \text{ kJ/mol} \end{aligned}$$

Question24

For a first order decomposition of a certain reaction, rate constant is given by the equation. $\log k\left(s^{-1}\right) = 7.14 - \frac{1 \times 10^4 \text{ K}}{T}$**. The activation energy of the reaction (in** kJmol^{-1} **) is**

$$\left(R = 8.3\text{JK}^{-1}\text{ mol}^{-1}\right)$$

Options:

A.

161.1

B.

171.1

C.

181.1

D.

191.1

Answer: D

Solution:

Given, $\log K\left(s^{-1}\right)=7.14-1\times10^{4}$ K

$R=8.3\mathrm{JK}^{-1}\ \mathrm{mol}^{-1}$ T
$E_a=?$

Using Arrhenius equation,

$\log k=\log A-\frac{E_a}{2.303RT}$

Comparing $\frac{E_a}{2.303R}=1\times10^{4}$

Calculating; $E_a=2.303\times R\times1\times10^{4}$

$=191.47$
≈ 191.1 kJ/mol

Question25

The source of an enzyme is malt and that enzyme converts X into Y. X and Y respectively are

Options:

A.

starch, maltose

B.

maltose, glucose

C.

proteins, peptides

D.

glucose, fructose

Answer: A

Solution:

Malt contain enzyme that converts starch into maltose specifically the enzyme diastase is responsible for breaking down starch into maltose

$$\underset{\substack{\text{Starch}\\(X)}}{(C_6H_{10}O_5)_n}\xrightarrow[\text{Amylose}]{\text{Diastase/}} n\times\underset{\substack{\text{Maltose}\\(Y)}}{C_{12}H_{22}O_{11}}$$

Question26

In the extraction of iron using blast furnace to remove the impurity (X), chemical (Y) is added to the ore. X and Y are respectively

Options:

A.

$SiO_2, MgCO_3$

B.

FeO, SiO_2

C.

$SiO_2, CaCO_3$

D.

$SiO_2, FeCO_3$

Answer: C

Solution:

Iron is made from its ore in a blast furnace. The iron ore has an impurity called X.

To remove X, another chemical, Y, is added to the mixture.

The chemical Y is calcium carbonate $(CaCO_3)$.

When Y is heated (Δ), it breaks down into calcium oxide (CaO) and carbon dioxide (CO_2):

$$\underset{(Y)}{CaCO_3} \xrightarrow{\Delta} CaO + CO_2$$

Next, calcium oxide (CaO) reacts with the impurity, silicon dioxide (SiO_2), which is X, to form a liquid waste called slag:

$$CaO + \underset{(\text{Impurity } X)}{SiO_2} \longrightarrow \underset{(\text{Slag})}{CaSiO_3}$$

So, X is silicon dioxide (SiO_2), and Y is calcium carbonate $(CaCO_3)$.

Question27

Thionyl chloride on reaction with white phosphorus gives a compound of phosphorus ' C ' which on hydrolysis gives an oxo acid ' O '. The correct statement about C and O are

I. Shape of ' C ' is pyramidal

II. ' O ' is a dibasic acid

III. ' O ' is a monobasic acid

IV. ' C ' on reaction with acetic acid given ' O '

Options:

A.

I and II only

B.

II and IV only

C.

I, III and IV only

D.

I, II and IV only

Answer: D

Solution:

Statment given in I, II and IV are correct, while III is incorrect. Its correct form is given in statement II.

The reaction involved are as follows.

$$P_4 + 8SOCl_2 \longrightarrow \underset{(C)}{4PCl_3} + 4SO_2 + 2S_2Cl_2$$

$$PCl_3 + 3H_2O \longrightarrow \underset{(D)}{H_3PO_3} + 3HCl$$

Question28

Which one of the following statements is not correct?

Options:

A.

Chlorine oxidises ferrous salts to ferric salts in acidic medium.

B.

Chlorine oxidises iodine to periodic acid in water.

C.

Chlorine acts as a bleaching agent due to oxidation.

D.

Chlorine is manufactured by Deacon's process.

Answer: B

Solution:

Option A: Chlorine oxidises ferrous salts to ferric salts in acidic medium.

- Ferrous (Fe^{2+}) has an oxidation state of +2. Ferric (Fe^{3+}) has an oxidation state of +3.
- Chlorine (Cl_2) is a strong oxidizing agent.
- In acidic medium, chlorine readily oxidizes Fe^{2+} to Fe^{3+}:

  $2Fe^{2+} + Cl_2 \rightarrow 2Fe^{3+} + 2Cl^-$
- This statement is **correct**.

Option B: Chlorine oxidises iodine to periodic acid in water.

- Iodine (I_2) has an oxidation state of 0.
- Periodic acid (HIO_4) has iodine in a +7 oxidation state.
- When chlorine reacts with iodine in the presence of water, it typically oxidizes iodine to iodic acid (HIO_3), where iodine is in a +5 oxidation state:

  $I_2 + 5Cl_2 + 6H_2O \rightarrow 2HIO_3 + 10HCl$
- To obtain periodic acid (HIO_4), a much stronger oxidizing agent than chlorine (like hot concentrated nitric acid, or by electrolysis, or certain peroxo compounds) is generally required, or specific conditions like strongly alkaline medium with hypochlorite. Simple "chlorine in water" does not usually produce periodic acid from iodine.
- Therefore, this statement is **incorrect**.

Option C: Chlorine acts as a bleaching agent due to oxidation.

- When chlorine dissolves in water, it forms hypochlorous acid (HOCl):

  $Cl_2 + H_2O \rightleftharpoons HCl + HOCl$

- Hypochlorous acid is a powerful oxidizing agent. It decomposes to release nascent oxygen ([O]), which oxidizes colored substances into colorless ones, thereby achieving bleaching.

  $HOCl \rightarrow HCl + [O]$

  Colored substance + [O] → Colorless substance

- Thus, the bleaching action of chlorine is indeed due to oxidation.

- This statement is **correct**.

Option D: Chlorine is manufactured by Deacon's process.

- Deacon's process is an industrial method for the production of chlorine. It involves the catalytic oxidation of hydrogen chloride gas with atmospheric oxygen, usually using copper(II) chloride as a catalyst at high temperatures:

  $4HCl(g) + O_2(g) \rightleftharpoons 2Cl_2(g) + 2H_2O(g)$

- While electrolysis of brine is the predominant modern method, Deacon's process is a known and historically significant method for chlorine manufacturing.

- This statement is **correct**.

Based on the analysis, Option B is the incorrect statement.

The final answer is $\boxed{B}$

Question29

Consider the following.

Assertion (A) Phosphorus can form both phosphorus (III) and phosphorus (V) chlorides but nitrogen cannot form nitrogen (V) chloride.

Reason (R) The electronegativity of nitrogen is more than that of phosphorus.

The correct answer is

Options:

A.

Both (A) and (R) are correct, (R) is the correct explanation of (A).

B.

(A) is correct, but (R) is not correct.

C.

Both (A) and (R) are correct, (R) is not the correct explanation of (A).

D.

(A) is not correct, but (R) is correct.

Answer: C

Solution:

Both the Assertion and the Reason are correct statements, but the Reason does not explain the Assertion properly.

Phosphorus can make both phosphorus(III) chloride and phosphorus(V) chloride because phosphorus has empty *d*-orbitals in its outer shell. This lets phosphorus use these orbitals to form more than four bonds, so it can form compounds like PCl_5 (phosphorus(V) chloride).

Nitrogen cannot form nitrogen(V) chloride (NCl_5) because nitrogen does not have *d*-orbitals in its valence shell. It can only make up to four bonds, so it cannot expand its octet to form five bonds like phosphorus can.

Therefore, the main reason why phosphorus can form PCl_5 but nitrogen cannot form NCl_5 is the availability of *d*-orbitals in phosphorus, not the difference in electronegativity.

Question30

$E^{\circ}_{M^{3}|M^{2+}}$ (in V) is highest for

Options:

A.

Fe

B.

Mn

C.

Cr

D.

V

Answer: B

Solution:

$E^{\circ}_{M^{3+}/M^{2+}}$ is highest for Mn i.e., $= +1.57$ V

It is due to stability of Mn^{2+} ion which has a half filled *d*-orbitals.

Question31

Arrange the following complexes in the increasing order of their spin only magnetic moment (in B.M)

I. $[Fe(CN)_6]^{4-}$

II. $[MnCl_4]^{2-}$

III. $[Mn(CN)_6]^{4-}$

IV. $[Cr(NH)_3)_6]^{3+}$

Options:

A.

II < IV < I < III

B.

III < II < I < IV

C.

I < IV < II < III

D.

I < III < IV < II

Answer: D

Solution:

The magnetic moment of a complex depends on the number of unpaired electrons present in it.

Formula: The spin-only magnetic moment is given by $\mu = \sqrt{n(n+2)}$ where n is the number of unpaired electrons.

Counting Unpaired Electrons for Each Complex:

I. $[Fe(CN)_6]^{4-}$: Here, $n = 0$. This means there are no unpaired electrons.

II. $[MnCl_4]^{2-}$: Here, $n = 5$. There are five unpaired electrons.

III. $[Mn(CN)_6]^{4-}$: Here, $n = 1$. There is one unpaired electron.

IV. $[Cr(NH_3)_6]^{3+}$: Here, $n = 3$. There are three unpaired electrons.

The complexes with more unpaired electrons will have higher magnetic moments.

So, the order from least to greatest magnetic moment is:

$I < III < IV < II$

Question32

Neoprene is the polymer of a monomer X. IUPAC name of X is

Options:

A.

1, 3-butadiene

B.

2-methyl-1, 3-butadiene

C.

2-iodo-1, 3-butadiene

D.

2-chloro-1,3-butadiene

Answer: D

Solution:

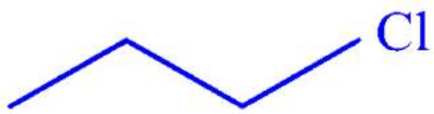
The monomer of neoprene is chloroprene. It's IUPAC name is, 2-chloro-1,3-butadiene.

Question33

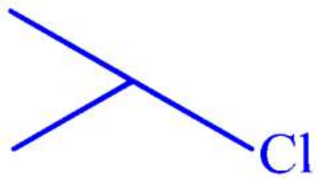
On prolonged heating with HI , glucose gives a compound ' C ' which can be obtained by Wurtz reaction using sodium metal and compound ' D ' . Identify ' D '

Options:

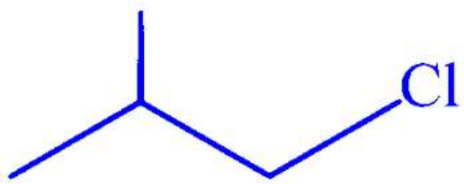
A.



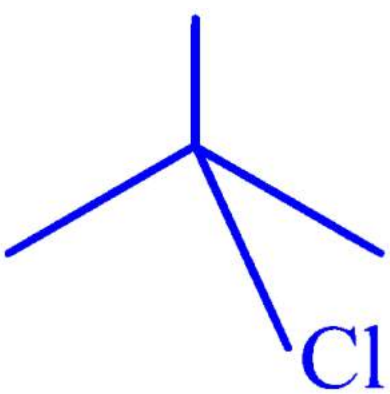
B.



C.

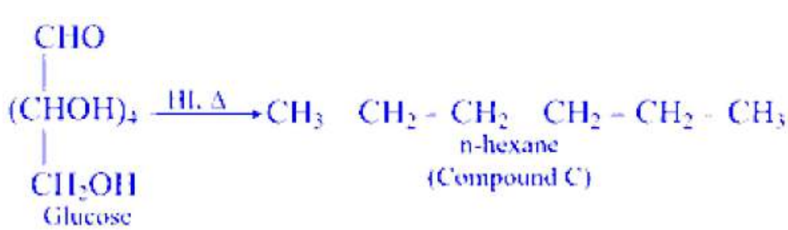


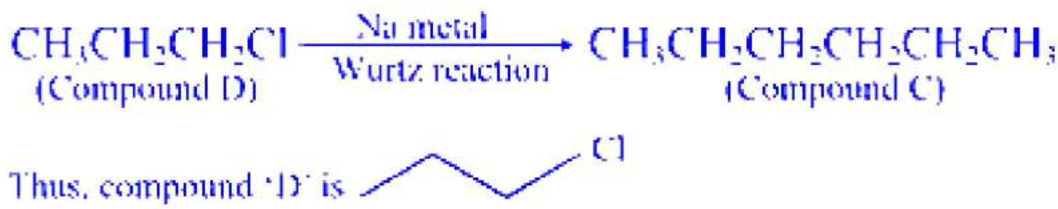
D.



Answer: A

Solution:





Question34

Match the following

List-I (Chemical)		List-II (Type)	
A	Bithionol	I	Artificial sweetener
B	Saccharin	II	Antifertility drug
C	Sodium benzoate	III	Antiseptic
D	Norethindrone	IV	Food preservative

The correct answer is

Options:

A.

A-III, B-I, C-IV, D-II

B.

A-II, B-I, C-IV, D-III

C.

A-III, B-II, C-IV, D-I

D.

A-IV, B-I, C-II, D-III

Answer: A

Solution:

- **A. Bithionol:** Bithionol is an antiseptic. It is commonly added to soaps to reduce its antiseptic properties.
  - Therefore, A matches with **III (Antiseptic)**.
- **B. Saccharin:** Saccharin is one of the oldest and most widely used artificial sweeteners.
  - Therefore, B matches with **I (Artificial sweetener)**.
- **C. Sodium benzoate:** Sodium benzoate is a common food preservative, particularly in acidic foods.
  - Therefore, C matches with **IV (Food preservative)**.
- **D. Norethindrone:** Norethindrone is a synthetic progestin that is used as a contraceptive (birth control pill). It is an antifertility drug.
  - Therefore, D matches with **II (Antifertility drug)**.

Combining these matches, we get:

A - III

B - I

C - IV

D - II

This combination matches **Option A**.

The final answer is $\boxed{A}$

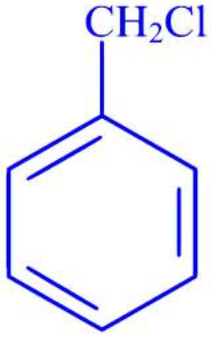
Question35

What is the product ' Z ' in the following reaction sequence?

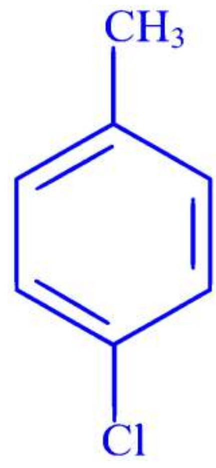
$$C_6H_5\,N_2Cl \xrightarrow[HCl]{Cu_2Cl_2} X \xrightarrow[\text{Na/dry ether}]{CH_3Cl} Y \xrightarrow[\text{Dark}]{Cl_2/Fe} Z$$

Options:

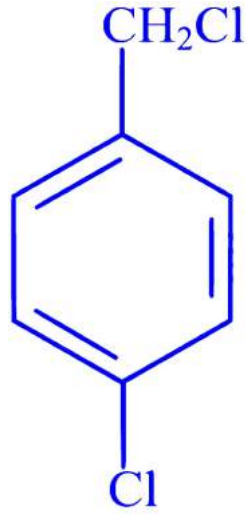
A.



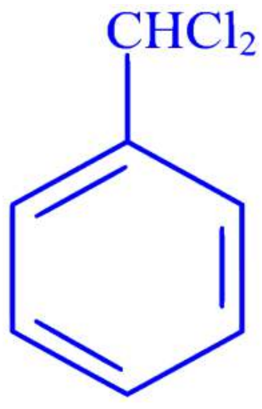
B.



C.



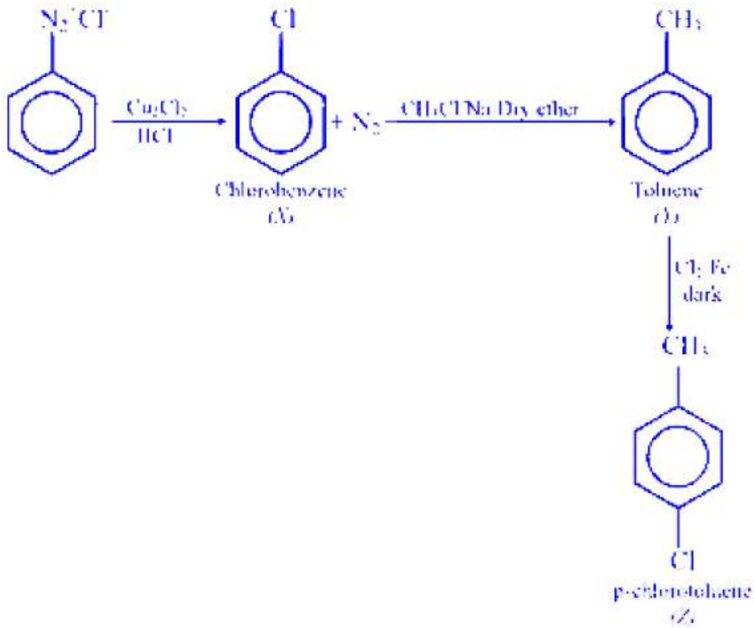
D.



Answer: B

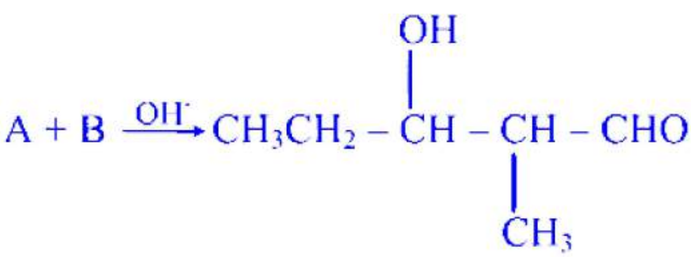
Solution:

The complete reaction sequence is



Question36

Identify the compounds A and B involved in the formation of given aldol



Options:

A.

$CH_3CH_2CH_2OH, CH_3CH_2CHO$

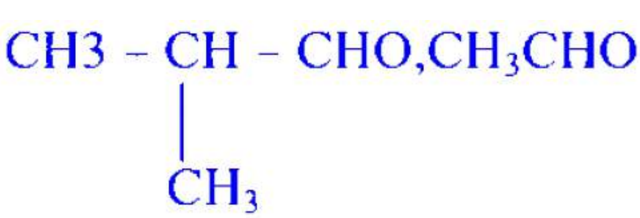
B.

CH_3COCH_3, CH_3CH_2CHO

C.

CH_3CH_2CHO, CH_3CH_2CHO

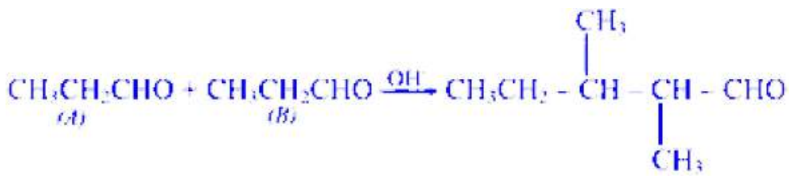
D.



Answer: C

Solution:

The complete reaction sequence is as follows,



Question37

In which of the following, intramolecular hydrogen bonding is present?

Options:

A.

Resorcinol

B.

Catechol

C.

Quinol

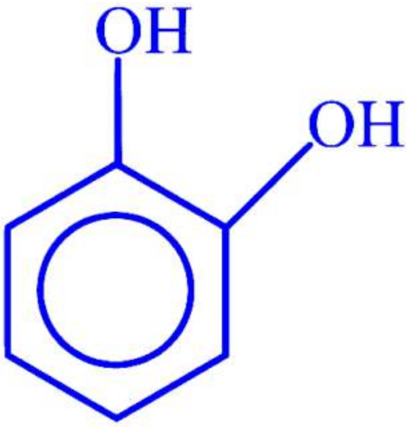
D.

o-cresol

Answer: B

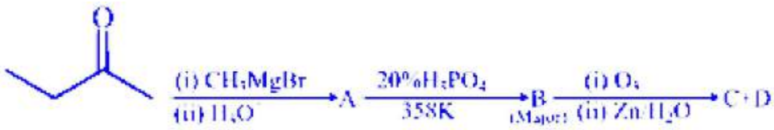
Solution:

Among the given options, intramolecular hydrogen bonding is present in catechol.



Question38

The products C and D are



Options:

A.

Ethanoic acid, ethanal

B.

Ethanol, propanone

C.

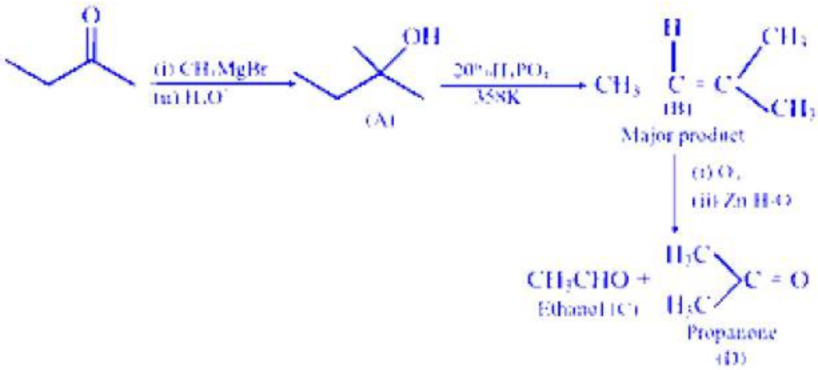
Ethanal, propanone

D.

Propanal, propanone

Answer: C

Solution:



Question39

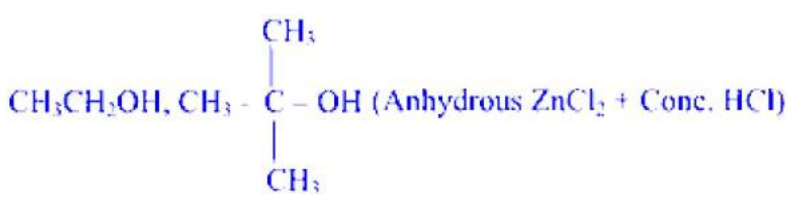
Identify the incorrect match with respect to compounds to be distinguished and reagent used

Options:

A.

$CH_3OH, CH_3CH_2OH \cdots - I_2 + NaOH$ solution)

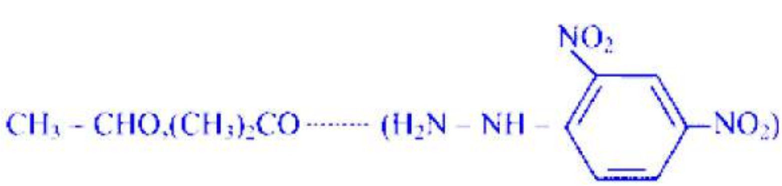
B.



C.

$CH_3 - C \equiv CH, CH_3 - C \equiv C - CH_3 \cdots - (Na)$

D.



Answer: D

Solution:

Incorrect match with respect to compounds to be distinguished and reagent used is given in option (d). It is because 2,4 -dinitrophenylhydrazine confirms the presence of carbonyl group ($C = O$) which is present in both aldehydes and ketones. Thus, this reagent cannot be used to distinguish given compounds.

Question40

The reagent which is used to distinguish primary, secondary and tertiary amines from the mixture is

Options:

A.

Fehling's reagent

B.

Tollen's reagent

C.

Lucas reagent

D.

Hinsberg's reagent

Answer: D

Solution:

Hinsberg's reagent is used to distinguish between primary, secondary and tertiary amines from a mixture.

Physics

Question1

The phenomenon of physics that deals with the constitution and structure of matter at the minute scales of atoms and nuclei is

Options:

A.

microscopic domain

B.

macroscopic domain

C.

classical physics

D.

thermodynamics

Answer: A

Solution:

Microscopic domain of physics is the phenomenon that deals with the constitution and structure of matter at the minute scales of atoms and nuclei.

Question2

If the length of a rod is measured as 830600 mm , then the number of significant figures in the measurement is

Options:

A.

5

B.

3

C.

6

D.

4

Answer: D

Solution:

The measured length is 830600 mm.

Because there is no decimal point in this number, the zeros at the end (the last two zeros) are not counted as significant figures.

So, only the digits 8, 3, 0, and 6 are significant.

Therefore, the number of significant figures in 830600 mm is 4.

Question3

A particle initially at rest is moving along a straight line with an acceleration of $2\ \mathrm{ms}^{-2}$. At a time of 3 s after the beginning of motion, the direction of acceleration is reversed. The time from the beginning of the motion in which the particle returns to its initial position is

Options:

A.

$(3+\sqrt{3})\mathrm{s}$

B.

$(2+\sqrt{2})\mathrm{s}$

C.

$3(2+\sqrt{2})\mathrm{s}$

D.

$2(3+\sqrt{3})s$

Answer: C

Solution:

Distance covered by the particle in first 3 seconds.

$$\begin{aligned} s &= ut+\frac{1}{2}at^2 \\ &= 0\times 3+\frac{1}{2}\times 2\times 3^2 \\ &= 9\ \mathrm{m} \end{aligned}$$

Velocity of particle after 3 s .

$v=u+at=0+2\times 3=6\ \mathrm{m/s}$

After $t=3$ s, direction of acceleration is reversed, so new acceleration,

$a' = -2 \text{ m/s}^2$

Since, particle returns to its initial position, thus,

$$\begin{aligned} & s_1 + s_2 = 0 \\ & s_2 = -s_1 = -s = -9 \text{ m} \\ \therefore \quad & s_2 = vt' + \frac{1}{2} a' (t')^2 \\ & -9 = 6t' + \frac{1}{2} \times (-2)(t')^2 \\ \Rightarrow \quad & (t')^2 - 6t' - 9 = 0 \\ \Rightarrow \quad & t' = \frac{-(-6) \pm \sqrt{36+36}}{2 \times 1} \\ & = 3 \pm 3\sqrt{2} \\ \Rightarrow \quad & t' = (3 + 3\sqrt{2}) \text{s} \end{aligned}$$

The total time from the beginning of the motion until the particle returns to its initial position,

$$\begin{aligned} T &= 3 + t' \\ &= 3 + 3 + 3\sqrt{2} = 6 + 3\sqrt{2} \\ &= 3(2 + \sqrt{2}) \text{s} \end{aligned}$$

Question4

If a body projected with a velocity of 19.6 ms^{-1} reaches a maximum height of 9.8 m , then the range of the projectile is

(Neglect air resistance)

Options:

A.

19.6 m

B.

78.4 m

C.

39.2 m

D.

9.8 m

Answer: B

Solution:

Step 1: Write the formula for maximum height

The formula for the highest point a projectile can reach is:

$H = \frac{u^2 \sin^2 \theta}{2g}$

where H is maximum height, u is the starting speed, θ is the angle of projection, and g is gravity.

Step 2: Put in the given values

We know $H = 9.8$ m, $u = 19.6$ ms^{-1}, and $g = 9.8$ ms^{-2}. We put these in the formula:

$9.8 = (19.6)^2 \frac{\sin^2 \theta}{2 \times 9.8}$

Step 3: Simplify to find $\sin^2 \theta$

Multiply both sides by 2×9.8:

$9.8 = 19.6 \times 19.6 \times \frac{\sin^2 \theta}{2 \times 9.8}$

$\Rightarrow 9.8 = 39.2 \sin^2 \theta$

Divide both sides by 19.6,

$\sin^2 \theta = \frac{1}{2}$

Step 4: Find the angle θ

$\sin \theta = \frac{1}{\sqrt{2}} = \sin 45°$, so

$\theta = 45°$.

Step 5: Write the formula for range R

The formula for how far the projectile lands (range) is:

$R = \frac{u^2 \sin 2\theta}{g}$

Step 6: Put in the numbers

Since $u = 19.6$ ms^{-1}, $\theta = 45°$, and $g = 9.8$ ms^{-2},

$R = \frac{(19.6)^2 \times \sin(2 \times 45°)}{9.8}$

$\sin(90°) = 1$, so

$R = \frac{(19.6)^2 \times 1}{9.8}$

$R = \frac{384.16}{9.8} = 39.2$ m

Question5

A force separately produces accelerations of 18 ms^{-2}, 9 ms^{-2} and 6 ms^{-2} in three bodies of masses P, Q and R respectively. If the same force is applied on a body of mass $P + Q + R$, then the acceleration of that body is

Options:

A.

3 ms^{-2}

B.

6 ms^{-2}

C.

2 ms^{-2}

D.

33 ms^{-2}

Answer: A

Solution:

Let m_P, m_Q, and m_R be the masses of the bodies P, Q, and R.

Step 1: Write the formula for mass using force and acceleration.

We know that $F = m \times a$. Rearranging, $m = \frac{F}{a}$.

Step 2: Find the mass of each body.

For P, $m_P = \frac{F}{18}$

For Q, $m_Q = \frac{F}{9}$

For R, $m_R = \frac{F}{6}$

Step 3: Add all the masses to get total mass.

$m_{PQR} = m_P + m_Q + m_R$

So, $m_{PQR} = \frac{F}{18} + \frac{F}{9} + \frac{F}{6}$

Find a common denominator: $m_{PQR} = \frac{F}{18} + \frac{2F}{18} + \frac{3F}{18} = \frac{6F}{18} = \frac{F}{3}$

Step 4: Find the acceleration of the combined mass with the same force.

Acceleration, $a_{PQR} = \frac{F}{m_{PQR}} = \frac{F}{(F/3)}$

The F cancels out: $a_{PQR} = 3\,\mathrm{m/s}^2$

Question6

A body of mass 500 g is falling from rest from a height of 3.2 m from the ground. If the body reaches the ground with a velocity of $6\ \mathrm{ms}^{-1}$, then the energy lost by the body due to air resistance is (Acceleration due to gravity $= 10\ \mathrm{ms}^{-2}$)

Options:

A.

14 J

B.

7 J

C.

21 J

D.

28 J

Answer: B

Solution:

Initial energy of body

$E_i = mgh = 0.5 \times 10 \times 3.2 = 16$ J

Final energy of the body

$E_f = \frac{1}{2}mv^2 = \frac{1}{2} \times 0.5 \times 6^2 = 9$ J

$\therefore$ Loss of energy due to air resistance

$\Delta U = U_i - U_f = 16 - 9 = 7$ J

Question7

A body of mass ' m ' moving with a velocity of ' v ' collides head on with another body of mass ' 2 m ' at rest. If the coefficient of restitution between the two bodies is ' e ', then the ratio of the velocities of the two bodies after collision is

Options:

A.

$\frac{1+e}{1-2e}$

B.

$\frac{1+2e}{1-e}$

C.

$\frac{1-e}{1+2e}$

D.

$\frac{1-2e}{1+e}$

Answer: D

Solution:

According to conservation of linear momentum

$$\Rightarrow \quad mv + 2m \times 0 = mv_1 + 2mv_2$$
$$\Rightarrow \quad v = v_1 + 2v_2 \qquad \ldots (i)$$

Coefficient of restitution

$$e = \frac{v_2 - v_1}{u_1 - u_2}$$
$$\Rightarrow \quad e \quad e = \frac{v_2 - v_1}{v - 0}$$
$$\Rightarrow \quad ev = v_2 - v_1$$
$$\Rightarrow \quad v_1 = v_2 - ev \qquad \ldots (ii)$$

From Eqs. (i) and (ii), we get

$$v = v_2 - ev + 2v_2$$
$$v = 3v_2 - ev$$
$$\Rightarrow \quad v(1+e) = 3v_2$$
$$\Rightarrow \quad v_2 = \frac{v(1+e)}{3} \qquad \ldots (iii)$$

Now, from Eq. (ii) and (iii), we get,

$$v_1 = \frac{v(1+e)}{3} - ev = \frac{v(1-2e)}{3}$$

$$\Rightarrow \quad v_1 = \frac{v(1-2e)}{3}$$

Thus, $\frac{v_1}{v_2} = \frac{1-2e}{1+e}$

Question8

A thin uniform circular disc of mass $\frac{10}{\pi^2}$ kg and radius 2 m is rotating about an axis passing through its centre and perpendicular to its plane. The work done to increase the angular speed of the disc from $90 \mathrm{rev}/\mathrm{min}$ to $120 \mathrm{rev}/\mathrm{min}$ is

Options:

A.

35 J

B.

70 J

C.

140 J

D.

210 J

Answer: B

Solution:

Step 1: Find the moment of inertia of the disc.

The moment of inertia (I) for a disc about its center is $I = \frac{1}{2}MR^2$

Here, the mass $M = \frac{10}{\pi^2}$ kg, and the radius $R = 2$ m.

So, $I = \frac{1}{2} \times \frac{10}{\pi^2} \times (2)^2 = \frac{1}{2} \times \frac{10}{\pi^2} \times 4 = \frac{20}{\pi^2}\ \mathrm{kg\,m}^2$

Step 2: Change revolutions per minute (rev/min) to angular speed (rad/s).

The first speed is 90 rev/min. To convert to radians per second:

$w_1 = 90 \times \frac{2\pi}{60} = 3\pi \text{ rad/s}$

The second speed is 120 rev/min: $w_2 = 120 \times \frac{2\pi}{60} = 4\pi \text{ rad/s}$

Step 3: Use the rotational kinetic energy formula to find work done.

The work done to increase speed equals the change in rotational kinetic energy:
$\text{Work Done} = \Delta KE = \frac{1}{2}I(w_2^2 - w_1^2)$

Plug in the values: $\text{Work} = \frac{1}{2} \times \frac{20}{\pi^2}\left[(4\pi)^2 - (3\pi)^2\right]$

Calculate:

$(4\pi)^2 = 16\pi^2$

$(3\pi)^2 = 9\pi^2$

So, $16\pi^2 - 9\pi^2 = 7\pi^2$

Therefore, $\text{Work} = \frac{1}{2} \times \frac{20}{\pi^2} \times 7\pi^2 = 10 \times 7 = 70 \text{ J}$

Question9

A solid cylinder of mass 2 kg , length 40 cm and radius 10 cm is placed in contact with a solid sphere of mass 0.5 kg and radius 10 cm such that the centres of the two bodies lie along the geometrical axis of the cylinder. The distance of the centre of mass of the system of two bodies from the centre of the sphere is

Options:

A.

27 cm

B.

15 cm

C.

24 cm

D.

18 cm

Answer: C

Solution:

Let the centre of the sphere be at origin ($x=0$).

Then, the centre of the cylinder lies at a distance of $x=0.3\ \mathrm{m}$ from the sphere centre.

CM of the system

$$
\begin{aligned}
X_{\mathrm{CM}} &= \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2} \\
&= \frac{2 \times 0.3 + 0.5 \times 0}{2 + 0.5} \\
X_{\mathrm{CM}} &= \frac{0.6}{2.5} = 0.24\ \mathrm{m}
\end{aligned}
$$

or $X_{\mathrm{CM}} = 24$ cm (From the centre of sphere)

Question10

If the amplitude of a damped harmonic oscillator becomes half of its initial amplitude in a time of 10 s , then the time taken for the mechanical energy of the oscillator to become half of its initial mechanical energy is

Options:

A.

2.5 s

B.

20 s

C.

10 s

D.

5 s

Answer: D

Solution:

Understanding the Amplitude Change:

The formula for how the amplitude (A) of a damped oscillator changes over time is: $A(t) = A_0 e^{-bt/2m}$ where A_0 is the starting amplitude, b is the damping constant, m is the mass, and t is time.

We are told the amplitude drops to half its original value in 10 seconds. So, $A(t) = \frac{A_0}{2}$ when $t = 10$ s.

Setting Up the Equation:

We set $\frac{A_0}{2} = A_0 e^{-10b/2m}$

Divide both sides by A_0 to get: $\frac{1}{2} = e^{-10b/2m}$

Take the natural logarithm of both sides: $\ln(2) = \frac{10b}{2m}$

So, $\frac{b}{m} = \frac{\ln(2)}{5}$

Now, How Long for Energy to Halve?

Mechanical energy E is proportional to the square of amplitude: $E \propto A^2$. It also decays exponentially, but at twice the rate: $E(t) = E_0 e^{-bt/m}$

We're looking for time when $E(t) = \frac{E_0}{2}$

So, $\frac{E_0}{2} = E_0 e^{-bt/m}$

Divide by E_0: $\frac{1}{2} = e^{-bt/m}$

Take the natural log: $\ln(2) = \frac{bt}{m}$

Plug in the Value of $fracbm$:

From before, $\frac{b}{m} = \frac{\ln(2)}{5}$, so substitute this into the energy equation:

$\ln(2) = \left[\frac{\ln(2)}{5}\right] t$

Divide both sides by $\frac{\ln(2)}{5}$: $t = 5$ seconds

Final Answer: It takes 5 seconds for the mechanical energy to become half of its original value.

Question11

A body is projected from the Earth's surface with a speed $\sqrt{5}$ times the escape speed (V_e). The speed of the body when it escapes from the gravitational influence of the Earth is

Options:

A.

2 V o

B.

V_e

C.

$3V_e$

D.

5 V_e

Answer: A

Solution:

Step 1: Use the conservation of energy

When an object moves in Earth's gravity, its total energy (kinetic plus potential) stays the same if no energy is lost.

Step 2: Write the energy equation

The equation is: $\frac{1}{2}mV_i^2 - \frac{GMm}{R} = \frac{1}{2}mV_f^2 + 0$

Here, V_i is the speed at the start, and V_f is the speed far away from Earth (where gravity is almost zero).

Step 3: Substitute the initial speed

The body is thrown with speed $V_i = \sqrt{5}V_e$, where V_e is escape velocity.

Step 4: Plug in values and simplify

$\frac{1}{2}m(\sqrt{5}V_e)^2 - \frac{gR^2m}{R} = \frac{1}{2}mV_f^2$

$\Rightarrow \frac{5}{2}mV_e^2 - gRm = \frac{1}{2}mV_f^2$

$\Rightarrow \frac{5}{2}V_e^2 - gR = \frac{1}{2}V_f^2$

Step 5: Express gR in terms of V_e

Remember, $V_e = \sqrt{2gR}$. Square both sides to get $V_e^2 = 2gR$.

So, $gR = \frac{V_e^2}{2}$.

Step 6: Substitute and solve for V_f

Plug $gR = \frac{V_e^2}{2}$ into the previous equation:

$\frac{5}{2}V_e^2 - \frac{V_e^2}{2} = \frac{1}{2}V_f^2$

$(5-1)\frac{V_e^2}{2} = \frac{1}{2}V_f^2$

$4\frac{V_e^2}{2} = \frac{1}{2}V_f^2$

Multiply both sides by 2:

$4V_e^2 = V_f^2$

Take square root:

$V_f = 2V_e$

Question12

A metal rod of area of cross-section $3\ \text{cm}^2$ is stretched along its length by applying a force of 9×10^4 N. If the Young's modulus of the material of the rod is $2 \times 10^{11}\text{Nm}^{-2}$, the energy stored per unit volume in the stretched rod is

Options:

A.

$13.5 \times 10^5 \text{Jm}^{-3}$

B.

$9 \times 10^5 \text{Jm}^{-3}$

C.

$225 \times 10^5 \text{Jm}^{-3}$

D.

$4.5 \times 10^5 \text{Jm}^{-3}$

Answer: C

Solution:

The energy stored per unit volume in a stretched rod

$$u=\frac{1}{2} \times \text{stress} \times \text{strain}$$
$$=\frac{1}{2} \times \text{stress} \times \frac{\text{stress}}{Y}$$
$$=\frac{1}{2} \times \frac{(\text{ stress })^2}{Y}=\frac{1}{2} \times \frac{\left(\frac{F}{A}\right)^2}{Y}$$
$$=\frac{1}{2} \times \frac{\left(\frac{9 \times 10^4}{3 \times 10^{-4}}\right)^2}{2 \times 10^{11}}=\frac{1}{2} \times \frac{9 \times 10^{16}}{2 \times 10^{11}}$$
$$=2.25 \times 10^5 \mathrm{Jm}^{-3}$$

Question13

An air bubble rises from the bottom to the top of a water tank in which the temperature of the water is uniform. The surface area of the bubble at the top of the tank is 125% more than its surface area at the bottom of the tank. If the atmospheric pressure is equal to the pressure of 10 m water column, then the depth of water in the tank is

Options:

A.

16.25 m

B.

27 m

C.

19 m

D.

23.75 m

Answer: D

Solution:

Step 1: Find Surface Area Relationship

The bubble's surface area at the top is 125% more than at the bottom. This means:

$$A_{\text{top}}=A_{\text{bottom}}+1.25 A_{\text{bottom}}=2.25 A_{\text{bottom}}$$

Step 2: Connect Surface Area and Volume

The volume (V) of a bubble is related to its surface area (A) by: $V \propto A^{3/2}$

Step 3: Express Volume Change

Let V_1 be the volume at the bottom, and V_2 at the top. Then,

$\frac{V_2}{V_1} = \left(\frac{A_2}{A_1}\right)^{3/2} = (2.25)^{3/2} = 3.375$

Step 4: Use Boyle's Law for Pressure and Volume

Since the water temperature is the same, Boyle's law applies: $p_1 V_1 = p_2 V_2$ $\frac{p_1}{p_2} = \frac{V_2}{V_1} = 3.375$

Step 5: Write Out Pressure at Bottom and Top

At the bottom, pressure is $p_1 =$ atmospheric pressure $+$ pressure from water column with height h Atmospheric pressure is equal to water pressure from $10\,\mathrm{m}$ column. So: $p_1 = 10 + h$ At the top, the pressure is just the atmospheric pressure: $p_2 = 10$

Step 6: Set Up the Equation and Solve for h

So, $\frac{p_1}{p_2} = \frac{10+h}{10}$ Set equal to 3.375: $\frac{10+h}{10} = 3.375$ $10 + h = 33.75$ $h = 23.75\,\mathrm{m}$

Question14

If W_1 is the work done in increasing the radius of a soap bubble from ' r ' to ' $2r$ ' and W_2 is the work done in increasing the radius of the soap bubble from ' $2r$ ' to ' $3r$ ', then $W_1 : W_2 =$

Options:

A.

$3:5$

B.

$1:1$

C.

$2:3$

D.

$3:4$

Answer: A

Solution:

To find the work done to make a soap bubble bigger, we use the formula:

$W = 2T \cdot \Delta A$

Step 1: Find Change in Surface Area for W_1

The surface area of a sphere is $4\pi r^2$. When the radius goes from r to $2r$, the change in area is:

$\Delta A = 4\pi(2r)^2 - 4\pi r^2$

This simplifies to:

$= 16\pi r^2 - 4\pi r^2 = 12\pi r^2$

Step 2: Find Work Done W_1

Plug the change in area into the work formula:

$W_1 = 2T \cdot 12\pi r^2 = 24\pi T r^2$

Step 3: Find Change in Surface Area for W_2

Now, change the radius from $2r$ to $3r$:

$\Delta A = 4\pi(3r)^2 - 4\pi(2r)^2$

$= 36\pi r^2 - 16\pi r^2 = 20\pi r^2$

Step 4: Find Work Done W_2

Plug in this change in area:

$W_2 = 2T \cdot 20\pi r^2 = 40\pi T r^2$

Step 5: Ratio $W_1 : W_2$

$W_1 : W_2 = 24 : 40 = 3 : 5$

Question15

To increase the length of a metal rod by 0.4% the temperature of the rod is to be increased by (Coefficient of linear expansion of the metal $= 20 \times 10^{-6} {}^\circ\text{C}^{-1}$)

Options:

A.

373 K

B.

473 K

C.

200 K

D.

100 K

Answer: C

Solution:

We know that

$$\begin{aligned} \Delta L &= L \propto \Delta T \\ \Rightarrow \Delta T &= \frac{\Delta L}{\Delta \alpha} = \left(\frac{\Delta L}{L}\right)\frac{1}{\alpha} \\ &= 0.004 \times \frac{1}{20 \times 10^{-6}} \\ &= 0.002 \times 10^{5} = 200 \text{ K} \end{aligned}$$

Question16

The power of a refrigerator that can make 15 kg of ice at $0^{\circ}\mathrm{C}$ from water at $30^{\circ}\mathrm{C}$ in one hour is

Options:

A.

6600 W

B.

1925 W

C.

2200 W

D.

4620 W

Answer: B

Solution:

Heat released from water to bring it at 0°C,

$$Q_1 = mc_W \Delta T = 15 \times 4200 \times 30$$
$$= 1.89 \times 10^6 \text{ J}$$

Heat released to convert water at 0°C to ice at 0°C,

$$Q_2 = mL_f = 15 \times 3.34 \times 10^5$$
$$= 5.01 \times 10^6 \text{ J}$$

Total heat released

$$Q = Q_1 + Q_2$$
$$= 1.89 \times 10^6 + 5.01 \times 10^6$$
$$= 6.9 \times 10^6 \text{ J}$$

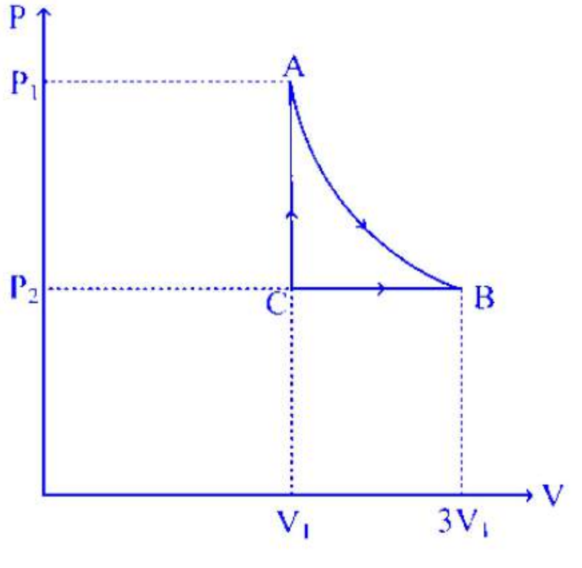
$\therefore$ Power of refrigerator, $P = \frac{Q}{t}$

$$= \frac{6.9 \times 10^6}{1 \times 60 \times 60}$$
$$= 1916.67 \text{ W}$$

Which is nearest to 1925 W

Question17

Three moles of an ideal gas undergoes a cyclic process $ABCA$ as shown in the figure. The pressure, volume and absolute temperature at points A, B and C are respectively (p_1, V_1, T_1), $(p_2, 3V_1, T_1)$ and (p_2, V_1, T_2). Then, the total work done in the cycle $ABCA$ is ($R =$ Universal gas constant).



Options:

A.

$RT_1[3\ln(3)+2]$

B.

$RT_1[3\ln(2)]$

C.

$3RT_1[\ln(3)]$

D.

$RT_1[3\ln(3)-2]$

Answer: D

Solution:

Given, process AB is isothermal Work done by the gas in isothermal process AB is,

$$\begin{aligned}
&= W_{AB} = nRT_1 \ln\left(\frac{V_2}{V_1}\right) \\
&= nRT_1 \ln\left(\frac{3V_1}{V_1}\right) \\
&= nRT_1 \ln 3 \\
&= 3RT_1 \ln(3)
\end{aligned}$$

Compression work of process BC on the gas is

$$\begin{aligned}
W_{BC} &= p\Delta V = p_2\left(V_1 - 3V_1\right) \\
&= -2p_2V_1 = -\frac{2}{3}p_2\left(3V_1\right) \\
&= -\frac{2}{3}\cdot 3RT_1 \quad \{\therefore p_2\left(3V_1\right) = 3RT_1\} \\
&= -2RT_1
\end{aligned}$$

Work done in process $CA = p\Delta V = 0$

Total work done in cyclic process $ABCA$ is,

$$\begin{aligned} W_{ABCA} &= W_{AB} + W_{BC} + W_{CA} \\ &= 3RT_1 \ln 3 - 2RT_1 \\ &= RT_1(3\ln(3-2) \end{aligned}$$

Question18

The pressure of a mixture of 64 g of oxygen, 28 g of nitrogen and 132 g of carbon dioxide gases in a closed vessel is p. Under isothermal conditions if entire oxygen is removed from the vessel, the pressure of the mixture of remaining two gases is

Options:

A.

p

B.

$\frac{3p}{2}$

C.

$\frac{p}{3}$

D.

$\frac{2p}{3}$

Answer: D

Solution:

Step 1: Calculate moles of each gas

$$n_{O_2} = \frac{64}{32} = 2 \text{ moles}$$
$$n_{N_2} = \frac{28}{28} = 1 \text{ mole}$$
$$n_{CO_2} = \frac{132}{44} = 3 \text{ moles}$$

Step 2: Add up all the moles for the total initial amount

$$n_i = n_{O_2} + n_{N_2} + n_{CO_2}$$
$$= 2 + 1 + 3 = 6 \text{ moles}$$

Step 3: Find out how many moles are left after removing all the oxygen gas

Only nitrogen (N_2) and carbon dioxide (CO_2) are left.

$$n_f = n_{N_2} + n_{CO_2} = 1 + 3 = 4 \text{ moles}$$

Step 4: Relate pressure and moles using the formula $pV = nRT$

Since temperature and volume stay the same, the pressure is directly proportional to the number of moles. So, $\frac{p_i}{p_f} = \frac{n_i}{n_f}$

The starting pressure is p, so

$\frac{p}{p_f} = \frac{6}{4}$

So, the final pressure is $p_f = \frac{2}{3}p$

Question19

A sound wave of frequency 210 Hz travels with a speed of 330 ms^{-1} along the positive X-axis. Each particle of the wave moves a distance of 10 cm between the two extreme points. The equation of the displacement function (s) of this wave is (x in metre, t in second)

Options:

A.

$s(x, t) = 0.10 \sin[4x - 1320t]\text{m}$

B.

$\text{s}(x, t) = 0.05 \sin[4x - 1320t]\text{m}$

C.

$s(x, t) = 0.05 \sin[1320x - 4t]\text{m}$

D.

$s(x,t) = 0.10 \sin[1320x - 4t] m$

Answer: B

Solution:

Step 1: Find the amplitude.

The total distance from one extreme point to the other is 10 cm. So the amplitude is half of this: $5 \text{ cm} = 0.05 \text{ m}$
.

Step 2: Find the angular frequency ((ω)).

The frequency f is 210 Hz.

Angular frequency: $\omega = 2\pi f = 2 \times \frac{22}{7} \times 210 = 44 \times 30 = 1320 \text{ rad/s}$.

Step 3: Find the wave number (k).

The speed v is 330 m/s.

Wave number: $k = \frac{\omega}{v} = \frac{1320}{330} = 4 \text{ rad/m}$.

Step 4: Write the equation of the wave.

The standard equation for a wave traveling along the positive X-axis is $s(x,t) = A \sin(kx - \omega t)$.

Here, $A = 0.05 \text{ m}$, $k = 4 \text{ rad/m}$, and $\omega = 1320 \text{ rad/s}$, so:

$s(x,t) = 0.05 \sin(4x - 1320t)$

Question20

A string vibrates in its fundamental mode when a tension T_1 is applied to it. If the length of the string is decreased by 25% and the tension applied is changed to T_2, the fundamental frequency of the string increases by 100%, then $\frac{T_2}{T_1} =$

(Linear density of the string is constant)

Options:

A.

$\frac{3}{8}$

B.

$\frac{2}{3}$

C.

$\frac{8}{9}$

D.

$\frac{9}{4}$

Answer: D

Solution:

Step 1: Formula for Fundamental Frequency

The basic formula for the frequency of a vibrating string is:

$$f = \frac{1}{2l}\sqrt{\frac{T}{\mu}}$$

Here, f is frequency, l is the length of the string, T is the tension, and μ is mass per unit length (which stays the same in this problem).

Step 2: Frequency with Initial Length and Tension

The first frequency, with length l and tension T_1 is:

$$f_1 = \frac{1}{2l}\sqrt{\frac{T_1}{\mu}}$$

Step 3: Frequency After Changing Length and Tension

The length of the string is shortened by 25%, so the new length is 75% of l, or $\frac{3}{4}l$. The new tension is T_2 and the new frequency becomes $2f_1$ (it increases by 100%):

$$2f_1 = \frac{1}{2(\frac{3}{4}l)}\sqrt{\frac{T_2}{\mu}}$$

Step 4: Set Up the Equation

Now, make it easier by expressing $2f_1$ in terms of the formula:

$$2f_1 = \frac{1}{(3/2)l}\sqrt{\frac{T_2}{\mu}}$$

Because splitting the denominator gives $2 \times \frac{1}{2(\frac{3}{4}l)} = \frac{1}{(3/2)l}$.

Step 5: Plug f_1 from Step 2 into the New Equation

Replace f_1 in the new formula using the old frequency formula:

$$2\left(\frac{1}{2l}\sqrt{\frac{T_1}{\mu}}\right) = \frac{1}{(3/2)l}\sqrt{\frac{T_2}{\mu}}$$

Step 6: Simplify the Equation

Multiply both sides to clear out the denominators:

$$\frac{1}{l}\sqrt{\frac{T_1}{\mu}} = \frac{2}{3l}\sqrt{\frac{T_2}{\mu}}$$

Multiply both sides by $3l$:

$$3\sqrt{\frac{T_1}{\mu}} = 2\sqrt{\frac{T_2}{\mu}}$$

Step 7: Get Rid of Square Roots by Squaring Both Sides

Square each side to eliminate the square roots:

$$9\frac{T_1}{\mu} = 4\frac{T_2}{\mu}$$

The μ cancels out:

$$9T_1 = 4T_2$$

Step 8: Find $\frac{T_2}{T_1}$

Divide both sides by $4T_1$ to solve for $\frac{T_2}{T_1}$:

$$\frac{T_2}{T_1} = \frac{9}{4}$$

Question21

An object of height 3.6 cm is placed normally on the principal axis of a concave mirror of radius of curvature 30 cm . If the object is at a distance of 10 cm from the principal focus of the mirror, then the height of the real image formed due to the mirror is

Options:

A.

5.4 cm

B.

3.6 cm

C.

1.8 cm

D.

2.7 cm

Answer: A

Solution:

Focal length of concave mirror

$f = \frac{R}{2} = \frac{-30}{2} = -15 \text{ cm}$

Object distance from the pole of the mirror

$u = -(f + 10) = -(-15 + 10) = 5 \text{ cm}$

According to question, for real image formed by a concave mirror, the object must be placed beyond the focal point. If the object is 10 cm from the principle focus and the focal length is 15 cm , the object distance from the pole is

$u = -(15 + 10) = -25 \text{ cm}$

Using mirror formula,

$$\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$$
$$\Rightarrow \frac{1}{-15} = \frac{1}{-25} + \frac{1}{v}$$
$$\Rightarrow \frac{1}{v} = \frac{1}{25} - \frac{1}{15} = \frac{-2}{75}$$
$$\therefore v = -37.5 \text{ cm}$$
$$\therefore m = \frac{h_i}{h_0} = \frac{-v}{u}$$
$$\Rightarrow \frac{h_i}{3.6} = \frac{-37.5}{-25}$$
$$\Rightarrow h_i = -5.4 \text{ cm}$$

The negative sign indicates that the image is real and inverted.

Question22

Monochromatic light of wavelength $6000 \overset{\circ}{\text{A}}$ incidents on a small angled prism. If the angle of the prism is 6°, the refractive indices of the material of the prism for violet and red lights are respectively 1.52 and 1.48 , then the angle of dispersion produced for this incident light is

Options:

A.

30°

B.

36°

C.

24°

D.

0°

Answer: D

Solution:

To find the angle of dispersion, we use the formula:

$\delta = (n_1 - n_2)A$

Here, n_1 and n_2 are the refractive indices for violet and red lights, and A is the prism's angle.

Plug in the values: $n_1 = 1.52$, $n_2 = 1.48$, $A = 6^{\circ}$.

$\delta = (1.52 - 1.48) \times 6 = 0.04 \times 6 = 0.24^{\circ}$

So, the angle of dispersion is 0.24°, which is about 0.2°.

Question23

In Young's double slit experiment, if the distance between 5th bright and 7th dark fringes is 3 mm , then the distance between 5th dark and 7th bright fringes is

Options:

A.

6 mm

B.

3 mm

C.

5 mm

D.

4 mm

Answer: C

Solution:

Step 1: Find the position equations for the fringes.

The position of the nth bright fringe is given by: $y_n = \frac{nD\lambda}{d}$

So, for the 5th bright fringe: $y_5 = \frac{5D\lambda}{d}$

The position of the nth dark fringe is: $y_n = \frac{(2n-1)D\lambda}{2d}$

For the 7th dark fringe: $y_7 = \frac{(2\times7-1)D\lambda}{2d} = \frac{13D\lambda}{2d}$

Step 2: Use the given distance between 5th bright and 7th dark fringes.

Distance between the 7th dark and 5th bright fringe: $y_7 - y_5 = \frac{13D\lambda}{2d} - \frac{5D\lambda}{d}$

First, make the denominators the same: $\frac{13D\lambda}{2d} - \frac{10D\lambda}{2d} = \frac{3D\lambda}{2d}$

The distance given in the question is 3 mm: $\frac{3D\lambda}{2d} = 3 \times 10^{-3}$ m

Step 3: Find the value of $\frac{D\lambda}{d}$.

Divide both sides by 3: $\frac{D\lambda}{2d} = 1 \times 10^{-3}$ Now multiply both sides by 2: $\frac{D\lambda}{d} = 2 \times 10^{-3}$ m So, $\frac{D\lambda}{d} = 2$ mm $\quad \ldots(i)$

Step 4: Find the positions needed for the second distance.

Position of the 7th bright fringe: $y_7' = \frac{7D\lambda}{d}$

Position of the 5th dark fringe: $y_5' = \frac{(2\times5-1)D\lambda}{2d} = \frac{9D\lambda}{2d}$

Step 5: Calculate the distance between 5th dark and 7th bright fringes.

$y_7' - y_5' = \frac{7D\lambda}{d} - \frac{9D\lambda}{2d}$ Make denominators the same: $\frac{14D\lambda}{2d} - \frac{9D\lambda}{2d} = \frac{5D\lambda}{2d}$

Step 6: Find the final answer using the value from earlier.

From (i), $\frac{D\lambda}{d} = 2$ mm, so: $\frac{5D\lambda}{2d} = \frac{5}{2} \times 2$ mm $= 5$ mm

Final Answer: Distance between 5th dark and 7th bright fringes is **5 mm**.

Question24

Four electric charges 2μC, Q, 4μC and 12μC are placed on X-axis at distance $x = 0, 1$ cm, 2 cm and 4 cm respectively. If the net force acting on the charge at origin is zero, then $Q =$

Options:

A.

-3.5μC

B.

$-1.75 \mu\mathrm{C}$

C.

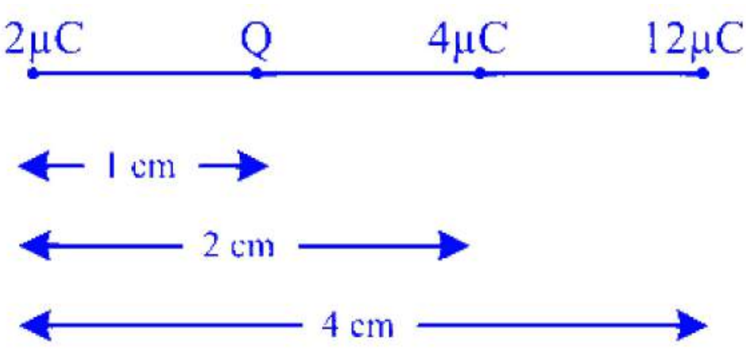
$-2.75 \mu\mathrm{C}$

D.

$-5.5 \mu\mathrm{C}$

Answer: B

Solution:



Net force on the charge at origin $=0$

$$\Rightarrow \frac{1}{4\pi\varepsilon_0}\left[\begin{array}{l}\frac{2\times10^{-6}Q}{(1\times10^{-2})^2}+\frac{2\times10^{-6}\times4\times10^{-6}}{(2\times10^{-2})^2}\\+\frac{2\times10^{-6}\times12\times10^{-6}}{(4\times10^{-2})^2}\end{array}\right]=0$$

$$\Rightarrow \frac{Q}{10^{-4}}+\frac{4\times10^{-6}}{4\times10^{-4}}+\frac{12\times10^{-6}}{16\times10^{-4}}=0$$

$$\Rightarrow Q+10^{-6}+\frac{3}{4}\times10^{-6}=0$$

$$\Rightarrow Q+\frac{7}{4}\times10^{-6}=0$$

$$\Rightarrow Q=-\frac{7}{4}\times10^{-6}\mathrm{C}=-\frac{7}{4}\mu\,\mathrm{F}=-1.75\mu\,\mathrm{F}$$

Question25

If a particle of mass 10 mg and charge $2 \mu\mathrm{C}$ at rest is subjected to a uniform electric field of potential difference 160 V , then the velocity acquired by the particle is

Options:

A.

9 ms^{-1}

B.

4 ms^{-1}

C.

6 ms^{-1}

D.

8 ms^{-1}

Answer: D

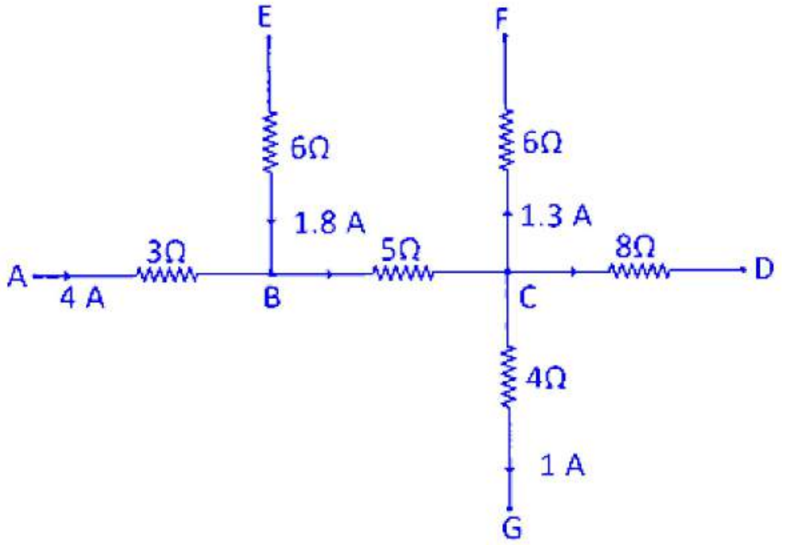
Solution:

According to work energy theorem,

$$W = q\Delta V$$
$$\text{KE}_f - \text{KE}_i = q\Delta V$$
$$\frac{1}{2}mv^2 - 0 = 2 \times 10^{-6} \times 160$$
$$\Rightarrow \frac{1}{2} \times 10^{-5} v^2 = 3.2 \times 10^{-4}$$
$$\Rightarrow v^2 = 64 \Rightarrow v = 8 \text{ m/s}$$

Question26

The potential difference between points C and D of the electrical circuit shown in the figure is



Options:

A.

28 V

B.

32 V

C.

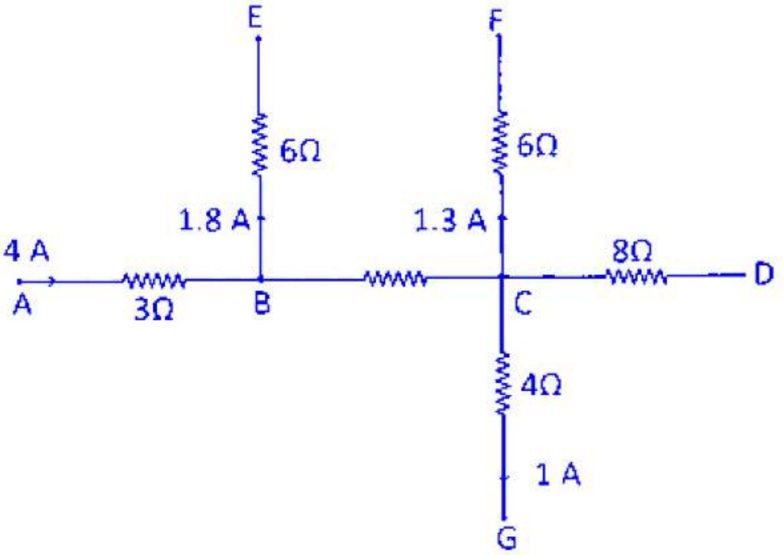
24 V

D.

20 V

Answer: A

Solution:



Using, $\Sigma I = 0$, at B,

$I_{BC} = 4 + 1.8 = 5.8$ A

Also,

At C,

$I_{CD} = 5.8 - 1.3 - 1 = 3.5$ A

So, potential difference between C and D is

$$V_{CD} = I_{CD} \times R_{CD}$$
$$= 3.5 \times 8 = 28.0 \text{ V}$$

Question27

The length of a potentiometer wire is 2.5 m and its resistance is 8Ω. A cell of negligible internal resistance and emf of 2.5 V is connected in series with a resistance of 242Ω in the primary circuit. The potential difference between two points separated by a distance of 20 cm on the potentiometer wire is

Options:

A.

1.6 mV

B.

4.8 mV

C.

6.4 mV

D.

3.2 mV

Answer: C

Solution:

Total resistance, $R = 242 + 8$

$= 250\Omega$

$\therefore$ Current in the primary circuit,

$I = \frac{E}{R} = \frac{2.5}{250} = 0.01 \text{ A}$

Potential drop across the potentiometer wire,

$$V' = I \times R_{\text{wire}} \quad = 0.01 \times 8$$
$$= 0.08 \text{ V}$$

$\therefore$ Potential gradient, $K = \frac{V}{L}$

$= \frac{0.08}{2.5} = 0.032 \text{ V/m}$

$\therefore$ Potential difference between two points separated by a distance of 20 cm ,

$$
\begin{aligned}
V'' &= K \times l = 0.032 \times 0.2 \\
&= 0.0064 \text{ V} \\
&= 6.4 \times 10^{-3} \text{ V} = 6.4\text{mV}
\end{aligned}
$$

Question28

The magnetic field due to a current carrying circular coil on its axis at a distance of $\sqrt{2}$ d from the centre of the coil is B. If d is the diameter of the coil, then the magnetic field at the centre of the coil is

Options:

A.

$18B$

B.

$27B$

C.

$3B$

D.

$9B$

Answer: B

Solution:

$$
B = \frac{\mu_0 I R^2}{2(R^2 + x^2)^{3/2}}
$$

$$
\begin{aligned}
here,\ x &= \sqrt{2}d = \sqrt{2}(2R) \qquad (\because d = 2R) \\
&= 2\sqrt{2}R
\end{aligned}
$$

$$
\begin{aligned}
\therefore \quad B &= \frac{\mu_0 I R^2}{2R^2 + (2\sqrt{2})^2\big]^{3/2}} \\
&= \frac{\mu_0 I R^2}{2[R^2 + 8R^2]^{3/2}}
\end{aligned}
$$

$$= \frac{\mu_0 I R^2}{2[(3R)^2]^{3/2}}$$
$$= \frac{\mu_0 I R^2}{2 \times (3R)^3}$$
$$B = \frac{\mu_0 I}{54R} = \frac{1}{27} \times \left(\frac{\mu_0 I}{2R}\right)$$
$$B = \frac{1}{27} \times B_C$$
$$\Rightarrow BC = 27B$$

Question29

A square coil of side 10 cm having 200 turns is placed in a uniform magnetic field of 2 T such that the plane of the coil is in the direction of magnetic field. If the current through the coil is 3 mA , then the torque acting on the coil is

Options:

A.

$12 \times 10^{-3}\text{Nm}$

B.

$24 \times 10^{-3}\text{Nm}$

C.

$6 \times 10^{-3}\text{Nm}$

D.

zero

Answer: A

Solution:

Torque, $\tau = NLAB\sin\theta$

$= 200 \times 3 \times 10^{-3} \times \left(10 \times 10^{-2}\right)^2 \times 2 \times \sin 90^\circ$

$= 12 \times 10^{-3}\ \mathrm{N-m}$

Question30

The magnetic field at a point P on the axis of a short bar magnet of magnetic moment M is B. If another short bar magnet of magnetic moment 2 M is placed on the first magnet such that their axes are perpendicular and their centres coincide. The resultant magnetic field at the point P due to both the magnets is

Options:

A.

$3B$

B.

$\sqrt{3}B$

C.

$\sqrt{2}B$

D.

$2B$

Answer: C

Solution:

On the axial position of bar magnet

$B_1 = \frac{\mu_0}{4\pi} \times \frac{2M}{r^3}$

Similarly,

$B_2 = \frac{\mu_0}{4\pi} \times \frac{M'}{r^3}$

here $m' = 2m$

$\therefore \quad B_2 = \frac{\mu_0}{4\pi} \cdot \frac{2m}{r^3}$

Since, $B_1 = B_2 = B$

$$\begin{aligned}\therefore B_{\text{resultant}} &= \sqrt{B_1^2 + B_2^2} \\ &= \sqrt{B^2 + B^2} = \sqrt{2}B\end{aligned}$$

Question31

A circular coil of area $3 \times 10^{-2}\ \mathrm{m}^2$, 900 turns and a resistance of 1.8Ω is placed with its plane perpendicular to a uniform magnetic field of 3.5×10^{-5} T. The current induced in the coil when it is rotated through 180° in half a second is

Options:

A.

2.1 mA

B.

1.8 mA

C.

1.5 mA

D.

2.7 mA

Answer: A

Solution:

Induced emf, $e = -N\frac{\Delta\phi}{\Delta t}$

$$= -\frac{900(\phi_2 - \phi)}{\Delta t}$$

$$= \frac{-900(BA\cos 180^\circ - BA\cos 0^\circ)}{\Delta t}$$

$$= -900\left(\frac{-2BA}{\Delta t}\right) = 900 \times \frac{2BA}{\Delta t}$$

$$= \frac{900 \times 2 \times 3.5 \times 10^{-5} \times 3 \times 10^{-2}}{0.5}$$

$$= 3.78 \times 10^{-3}\ \text{V}$$

$$\therefore \quad F = \frac{e}{R} = \frac{3.78 \times 10^{-3}}{1.8}$$

$$= 2.1 \times 10^{-3}\ \text{A} = 2.1\ \text{mA}$$

Question32

An electric bulb, an open coil inductor, an AC source and a key are all connected in series to form a closed circuit. They key is closed and after some time an iron rod is inserted into the interior of the inductor, then

Options:

A.

the glow of the bulb increases.

B.

the glow of the bulb remains unchanged.

C.

the glow of the bulb decreases.

D.

the bulb does not glow.

Answer: C

Solution:

When iron rod is inserted into interior of the inductor, then its self inductance increases.

Since, $X_L = \omega L$

i.e., $X_L \propto L$

Thus, due to increase in the value of L, X_L also increases.

Hence, current decreases.

Thus, the glow of the bulb decreases.

Question33

If the rate of change in electric flux between the plates of a capacitor is $9\pi \times 10^3 \mathrm{Vms}^{-1}$, then the displacement current inside the capacitor is

Options:

A.

0.36μ A

B.

0.25μ A

C.

3.14μ A

D.

4μ A

Answer: B

Solution:

Displacement current

$$I_D = \varepsilon_0 \frac{d\phi}{dt}$$
$$= 8.854 \times 10^{-12} \times 9\pi \times 10^3$$
$$= 0.25 \times 10^{-6} \text{ A}$$
$$= 0.25\mu \text{ A}$$

Question34

20 kV electrons can produce X- rays with a minimum wavelength of

Options:

A.

$0.248\overset{o}{A}$

B.

$0.41\overset{o}{A}$

C.

0.099 nm

D.

0.062 nm

Answer: D

Solution:

To find the smallest (minimum) wavelength of X-rays made by 20 kV electrons, use this formula:

$\lambda_{min} = \frac{hc}{E} = \frac{hc}{eV}$ where:
h = Planck's constant = 6.626×10^{-34} Js
c = speed of light = 3×10^{8} m/s
e = charge of electron = 1.6×10^{-19} C
V = voltage = 20,000 V = 20×10^{3} V

Plug in the numbers:

$\lambda_{min} = \frac{6.626 \times 10^{-34} \times 3 \times 10^{8}}{1.6 \times 10^{-19} \times 20 \times 10^{3}}$

Solve the calculation:

$\lambda_{min} = 0.062 \times 10^{-9}$ m

We can also write this as:

$\lambda_{min} = 0.062$ nm

Question35

The ratio of wavelengths of second line in Balmer series and the first line in Lyman series of hydrogen atom is

Options:

A.

2 : 1

B.

9 : 4

C.

4 : 1

D.

3 : 2

Answer: C

Solution:

$\frac{1}{\lambda} = R\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$

For the second line of Balmer series,

$$n_1 = 2 \text{ and } n_2 = 4$$
$$\therefore \frac{1}{\lambda_B} = R\left(\frac{1}{2^2} - \frac{1}{4^2}\right)$$
$$= R\left(\frac{1}{4} - \frac{1}{16}\right) = \frac{3R}{16}$$

$$\Rightarrow \lambda_B = \frac{16}{3R}$$

For the first line of Lyman series,

$n_1 = 1$ and $n_2 = 2$

$$\therefore \quad \frac{1}{\lambda_L} = R\left(\frac{1}{1^2} - \frac{1}{2^2}\right) = R\left(1 - \frac{1}{4}\right) = \frac{3R}{4}$$
$$\therefore \quad \lambda_L = \frac{4}{3R}$$
$$\therefore \quad \frac{\lambda_B}{\lambda_L} = \frac{\frac{16}{3R}}{\frac{4}{3R}} = \frac{4}{1}$$
$$\therefore \lambda_B : \lambda_L = 4 : 1$$

Question36

A radioactive material of half-life 2.5 hours emits radiation that is 32 times the safe maximum level. The time (in hours) after which the

material can be handled safely is

Options:

A.

10

B.

25

C.

5

D.

12.5

Answer: D

Solution:

The safe level of radiation is 32 times less than what the material gives off now.

Each time a half-life (2.5 hours) passes, the radiation becomes half of what it was before.

We can write this as: $\left(\frac{1}{2}\right)^n = \frac{1}{32}$ where n is the number of half-lives needed until the radiation is safe.

We know $\frac{1}{32} = \left(\frac{1}{2}\right)^5$. So, $n = 5$.

This means it takes 5 half-lives to reach a safe level.

Calculating total time:

$\frac{t}{T_{1/2}} = 5 \Rightarrow t = 5 \times T_{1/2}$

Each half-life is 2.5 hours, so total time is:

$t = 5 \times 2.5 = 12.5$ hours

Question37

If the number of uranium nuclei required per hour to produce a power of 64 kW is 7.2×10^{18}, then the energy released per fission is

Options:

A.

0.64×10^{-10} J

B.

3.2×10^{-13} J

C.

0.32×10^{-10} J

D.

3.2×10^{-10} J

Answer: C

Solution:

$$
\begin{aligned}
& E = P \times t \\
& = 64 \times 10^3 \times (60 \times 60) \\
& = 64 \times 10^3 \times 3.6 \times 10^3 \\
& = 2.3 \times 10^8 \text{ J} \\
& \therefore \text{ Energy released per fission } = \frac{E}{N} \\
& \quad = \frac{2.304 \times 10^8}{7.2 \times 10^{18}} = 0.32 \times 10^{-10} \text{ J}
\end{aligned}
$$

Question38

According to a graph drawn between the input and output voltages of a transistor connected in common emitter configuration, the region in which transistor acts as a switch is

Options:

A.

cutoff or saturation region

B.

active region

C.

active or saturation region

D.

cutoff or active region

Answer: A

Solution:

To use transistor as a switch in a common emitter configuration, the transistor operates in cutoff and saturation region.

In cutoff region, the transistor is "off" and acts like an open switch, allowing no current to flow between the collector and emitter.

In saturation region, the transistor is "on" and acts like a closed switch (short circuit) between the collector and emitter, allowing maximum current to flow.

In active region, transistor works as an amplifier.

Question39

If the energy gap of a semiconductor used for the fabrication of an LED is nearly 1.9 eV , then the color of the light emitted by the LED is

Options:

A.

white

B.

red

C.

green

D.

blue

Answer: B

Solution:

$$\begin{aligned}\lambda &= \frac{hc}{E} \\ &= \frac{6.62 \times 10^{-34} \times 3 \times 10^{8}}{1.9 \times 1.6 \times 10^{-19}} \\ &= 6.53 \times 10^{-7} \text{ m} \\ &\simeq 653 \text{ nm}\end{aligned}$$

The visible light spectrum ranges approximately 400 nm (violet) to 700 nm (red). A wavelength of 653 nm falls with in the red part of the spectrum. Therefore, the colour of the light emitted by the LED is red.

Question40

When the receiving antenna is on the ground, the range of a transmitting antenna of height 980 m is (Radius of the Earth $= 6400$ km)

Options:

A.

56 km

B.

112 km

C.

72.4 km

D.

224 km

Answer: B

Solution:

$$\begin{aligned}&\text{Range, } d = \sqrt{2Rh} \\ &= \sqrt{2 \times 6400 \times 10^{3} \times 980} \\ &= 112 \times 10^{3} \text{ m} = 112 \text{ km}\end{aligned}$$
